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Brief paper

Target-point formation control☆

Shaoshuai Mou [a], Ming Cao [b], A. Stephen Morse [c]

[a] School of Aeronautics and Astronautics, Purdue University, USA

[b] Faculty of Mathematics and Natural Sciences, ENTEG, University of Groningen, The Netherlands

[c] Department of Electrical Engineering, Yale University, USA

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ABSTRACT

In this paper a new distributed feedback strategy is proposed for controlling a rigid, acyclic formation of kinematic point-modeled mobile autonomous agents in the plane. The strategy makes use of a new concept called a "target point" and is applicable to any two-dimensional, acyclic formation whose underlying directed graph can be generated by a sequence of Henneberg vertex additions. It is shown that the method can cause a group of agents starting in any given initial positions in the plane to move into a prescribed formation exponentially fast provided the formation's designated leader and first follower start in different positions.

1. Introduction

Multi-agent formations have been employed to perform tasks such as surveillance (Diehl, Satharishi, Hampshire, & Khosla, 1999), exploration (Burgard, Moors, Fox, Simmons, & Thrun, 2000), search and rescue (Shiroma, Chiu, Sato, & Matsuno, 2005), ocean sampling (Leonard et al., 2007) and space missions (Krieger, Hajnsek, Papathanssiou, Younis, & Moreira, 2010). By a *multi-agent formation* is meant a collection of autonomous agents in which the distance between every pair of the agents is a prescribed constant as time evolves. Assuming the kinematic model of each agent is a single integrator (Krick, Broucke, & Francis, 2009), double-integrator (Olfati-Saber & Murray, 2002) or nonholonomic (Desai, Ostrowski, & Kumar, 2001), the problem of distributed formation control is to maintain a multi-agent formation by choosing a control input for each agent using the agent's local sensed information about its neighbors. The local measurement of each agent can be range only (Cao, Yu, & Anderson, 2011), bearing only (Basiri, Bishop, & Jensfelt, 2010) or relative positions (Dorfler & Francis, 2010). When a multi-agent formation is rigid (Anderson, Yu, Fidan, & Hendrickx, 2008; Asimow & Roth, 1979), the formation can be achieved by maintaining the desired distances between some chosen pairs of agents. If each such distance in a formation is maintained by both associated agents, the formation is *undirected*; otherwise, it is *directed*, in which the agent assigned with the task of maintaining the desired distance is called a *follower* and the other agent is correspondingly called its *leader*.

In the research line of controlling undirected formations, perhaps the most comprehensive distributed method based on rigidity is the gradient control proposed in Krick et al. (2009). It has been shown by center manifold theory that the gradient control locally stabilizes a large class of rigid undirected formations. Recent studies in Belabbas, Mou, Morse, and Anderson (2012), Helmke, Mou, Sun, and Anderson (2014) and Mou, Morse, Belabbas, Sun, and Anderson (in press) have revealed that undirected formations are problematic in the sense that they will rotate in the plane under the gradient control if there exists inconsistency in neighboring agents' distance measurements. Although progress to fix such an issue has recently been made in Mou, Morse, and Anderson (2014), Mou and Morse (2014) and Marina, Cao, and Jayawardhana (2015), how to achieve robustness in controlling rigid undirected formations under inconsistent measurements between neighboring agents is still an open problem. Growing interests have then been given to directed formations because of their known additional advantage of less usage of sensing

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E-mail addresses: mous@purdue.edu (S. Mou), m.cao@rug.nl (M. Cao), as.morse@yale.edu (A. Stephen Morse).

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and communication capabilities. Along this direction, sufficient and necessary graphical conditions have been derived for driving directed formations to a rendezvous point or a line (Lin, Francis, & Maggiore, 2005); switching has been introduced in the control design to stabilize a class of minimal persistent directed formations in Fidan, Gazi, Zhai, Cen, and Karatas (2013) and Sandeep, Fidan, and Yu (2006); and a virtual leader strategy has been employed in Ogren, Egerstedt, and Hu (2002). However, controlling directed formations in a distributed manner is challenging even for a four-agent formation consisting of two cycles (Belabbas, 2013). Instead of attacking the problem of controlling directed formations in general, researchers have recently focused on the class of minimally rigid acyclic formations (Baillieul & Suri, 2003; Ding, Yan, & Lin, 2010; Fidan et al., 2013; Mou et al., 2011; Sandeep et al., 2006).

The authors of Cao, Morse, Yu, Anderson, and Dasgupta (2011) have shown that the gradient control is able to stabilize an acyclic triangular formation if the three agents are not initialized collinear. Otherwise, the formation will drift to infinity with agents remaining in collinear positions. One reason for the difficulty in the global analysis of gradient control is the possibility of the formation converging to some equilibria determined by the local minima of the associated potential functions calculating the gradient. What this implies in terms of the evolution of the formation shape dynamics is that there are initial agent positions that may lead to incorrect formation shapes when agents are under gradient control. For example, a four-agent minimally rigid acyclic formation will fail to converge to its desired shape if its three-agent sub-formation starts with a wrong orientation. Thus there is an urgent need to look at other types of control to overcome these problems and complements the existing gradient control. And this is exactly the aim of this paper.

In this paper we utilize the idea of having the follower agents tracking their "target points", which are the desired relative positions for the follower agent to move to, to develop another type of formation control. Note that in a minimally rigid acyclic formation there is one agent called the *global leader*, which has no leader to follow, one agent called the *first follower*, which only follows the global leader, and each of all the other agents has exactly two leaders (Hendrickx, Anderson, Delvenne, & Blondel, 2007). In the following we will consider the case when an agent has one leader and the case when an agent has two leaders, separately, and propose a distributed control based on target points for both cases. Such analytical results will then be further used to prove the main result of this paper, namely, the proposed target-point control is able to drive minimally rigid acyclic formations to converge to their desired shapes as long as the global leader and the follower are not coincident.

2. Problem formulation

Consider the class of minimally rigid acyclic formations in the plane consisting of $n \geq 2$ mobile autonomous agents. It has been shown in Hendrickx et al. (2007) that any such an n-agent formation can be generated by the Henneberg vertex addition operations. And in addition that the agents can be labeled such that starting from a two-agent directed formation in which agent 2 follows agent 1, one adds in sequence an agent i, $3 \leq i \leq n$, which chooses two agents $i, j \in \{1, 2, \ldots, i-1\}$ as its leaders. We further suppose the distance between a follower and any of its leaders and the distance between the two leaders of agent i, $i = 3, 4, \ldots, n$ are positive. We call this class of formations *vertex-addition formations*. For example, the formations in Fig. 1(a)–(c) are vertex-addition formations while the one in Fig. 1(d) is not.

Assume each agent's motion in the plane is described by a simple kinematic point model

$$\dot{x}_i = u_i, \quad i = 1, 2, \ldots, n, \tag{1}$$

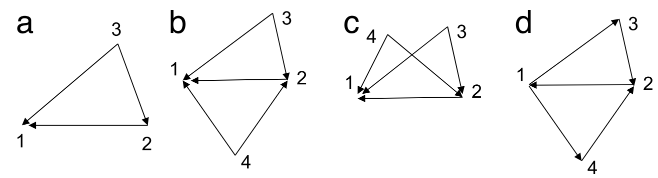

Fig. 1. Minimally rigid directed formations.

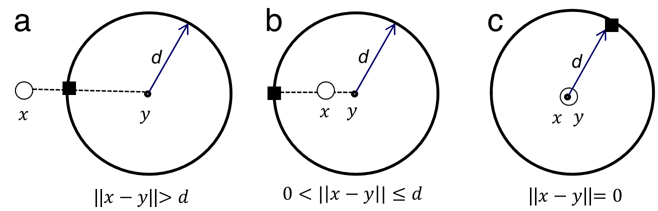

Fig. 2. Target points in the one-leader case.

where $x_i \in \mathbb{R}^2$ denotes the position of agent i and u_i denotes agent i's control input. Suppose each agent i's local measurements are $x_i - x_j$, the relative position away from each of its leader j. The goal of distributed formation control is to design u_i using agent i's local measurements such that the formation converges to its *desired shape*, that is, the distances between agent i, $i = 2, \ldots, n$ and each of its leader converge to the prescribed constants respectively, and each agent i, $i = 3, 4, \ldots, n$ and its two leaders are in desired clockwise or counter-clockwise orientation.

3. Target-point control

In a vertex-addition formation, a follower agent has at most two leaders. In the following we will devise target-point controls for the follower for both the one-leader case and the two-leader case.

3.1. One-leader case

Consider a two-agent directed formation in the plane with one follower and one leader, whose positions are denoted by $x, y \in \mathbb{R}^2$, respectively. Suppose the follower's motion in the plane is modeled by $\dot{x} = u$. The goal in this case is to choose u in terms of the follower's local measurement $x - y$ such that $\|x - y\|$ converges to the desired constant d. To accomplish this we define the following *target point*

$$\tau(x, y) = \begin{cases} x + \dfrac{\|y - x\| - d}{\|y - x\|}(y - x), & x \neq y; \\ y + \begin{bmatrix} d & 0 \end{bmatrix}', & x = y, \end{cases}$$

which is indicated by a black square in Fig. 2.

Note that the target point $\tau(x, y)$ is such that $\|\tau(x, y) - y\| = d$. One way to move the follower to keep d distance away from its leader is to drive x to converge to $\tau(x, y)$. Inspired by this observation, we choose

$$u = -\lambda(x - \tau(x, y)),$$

where λ is a non-negative parameter to be designed such that the control u is continuous. One choice for λ is

$$\lambda = \|y - x\|(\|y - x\| + d),$$

which leads to the following *target-point control* in one-leader case

$$u = -(\|x - y\|^2 - d^2)(x - y). \tag{2}$$

Note that the target-point control (2) is distributed in the sense that its implementation only requires the follower's local measurement $x - y$, and nothing else.