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Optimal control of stochastic functional neutral differential equations with time lag in control[☆]

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Abstract

In this work, we consider an optimal control problem of a class of stochastic differential equations driven by additive noise with aftereffect appearing in control. We develop a semigroup theory of the driving deterministic neutral system and identify explicitly the adjoint operator of the corresponding infinitesimal generator. We formulate the time delay equation under consideration into an infinite dimensional stochastic control system without time lag by means of the adjoint theory established. Consequently, we can deal with the associated optimal control problem through the study of a Hamilton–Jacob–Bellman (HJB) equation. Last, we present an example whose optimal control can be explicitly determined to illustrate our theory.

1. Introduction

Many physical phenomena can be modeled by stochastic dynamical systems whose evolution in time is governed by random forces as well as intrinsic dependence of the state on a finite period of its past history. In those dynamical systems with aftereffect,

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neutral-type systems, deterministic or stochastic, play a special role in both theory and practical application. In the meanwhile, there exist extensive researches devoted to those stochastic control systems with time delay in the controller's design, e.g., see Hale and Lunel [6], Kolmanovskii and Myshkis [10], Mao [12], Salamon [13] and references therein.

To present a motivational example, let us mention the transmission line equation (see [1])

$$\begin{cases} a\dfrac{\partial y(t,\xi)}{\partial t} + \dfrac{\partial z(t,\xi)}{\partial \xi} = 0, \\ \dfrac{1}{a}\dfrac{\partial z(t,\xi)}{\partial t} + \dfrac{\partial y(t,\xi)}{\partial \xi} = 0, \quad t \geq 0, \quad \xi \in [0,1], \end{cases} \tag{1.1}$$

with initial condition $y(0,\xi) = y_0(\xi)$, $z(0,\xi) = z_0(\xi)$ where $a > 0$ is a constant. Let

$$\begin{pmatrix} w_1(t,\xi) \\ w_2(t,\xi) \end{pmatrix} = \begin{pmatrix} 1/\sqrt{a} & \sqrt{a} \\ -1/\sqrt{a} & \sqrt{a} \end{pmatrix} \begin{pmatrix} z(t,\xi) \\ y(t,\xi) \end{pmatrix}, \tag{1.2}$$

then (w_1, w_2) satisfies the equation

$$\frac{\partial}{\partial t}\begin{pmatrix} w_1(t,\xi) \\ w_2(t,\xi) \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \partial w_1(t,\xi)/\partial \xi \\ \partial w_2(t,\xi)/\partial \xi \end{pmatrix}. \tag{1.3}$$

It may be shown that there exists a unique solution (w_1, w_2) to Eq. (1.3). In practice, there are boundary noise conditions which are determined by lump terminal networks. One possible set of these kinds of conditions is like

$$dy(t,0) + r_1 y(t,0)dt + r_2 dw(t) = z(t,0)dt, \; z(t,1) = 0, \tag{1.4}$$

where $r_i \in \mathbb{R}$, $i = 1, 2$, and $w(t)$, $t \geq 0$, is a real standard Brownian motion on some probability space $(\Omega, \mathscr{F}, \mathbb{P})$. Upon consideration of a control problem of Eq. (1.1), it has been noticed that there are occasions when a controller with time delay may quench the oscillation, yielding a smooth and fast transient response. This consideration basically leads to a controlled system with a retarded boundary control of the form

$$dy(t,0) + r_1 y(t,0)dt + bv(t-r)dt + r_2 dw(t) = z(t,0)dt, \; z(t,1) = 0, \tag{1.5}$$

where $b \in \mathbb{R}$, $r > 0$ and $v(t)$, $t \geq -r$, is the control for the overall system. Let us define $x_1(t) = w_1(t,0)$ and $x_2(t) = w_2(t,1)$, $t \geq 0$. Upon substitution of these new terms into Eq. (1.5), one can derive the following stochastic delay differential equation of neutral type

$$\begin{cases} d(x_1(t) + x_2(t-r)) + \alpha(x_1(t) + x_2(t-r))dt = \beta(x_1(t) - x_2(t-r))dt + \gamma v(t-r)dt \\ \quad + \delta dw(t), \quad t \geq 0, \\ x_1(t-r) - x_2(t) = 0, \qquad t \geq 0, \end{cases} \tag{1.6}$$

where $\alpha, \beta, \gamma, \delta$ are constants which can be determined by r_1, r_2 and a. Basically, we have from Eq. (1.6) that $x_2(t) = x_1(t-r)$, $t \geq 0$, and let $u(t) = v(t+r)$, $t \geq -2r$. Substitution of these into Eq. (1.6) yields further a stochastic neutral differential equation with respect to $x(t) \equiv x_1(t)$, $t \geq -2r$,

$$d(x(t) + x(t-2r)) = a_0 x(t)dt + a_1 x(t-2r)dt + b_0 u(t-2r)dt + c_0 dw(t), \tag{1.7}$$

where a_0, a_1, b_0, $c_0 \in \mathbb{R}$, with initial condition

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