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A new direct method based on the Chebyshev cardinal functions for variable-order fractional optimal control problems

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Abstract

In this paper, a new direct method based on the Chebyshev cardinal functions is proposed to solve a class of variable-order fractional optimal control problems (V-OFOCPs). To this end, a new operational matrix (OM) of variable-order (V-O) fractional derivative in the Caputo sense is derived for these basis functions and is used to obtain an approximate solution for the problem under study. In the proposed method, the state and the control variables are expanded in terms of the Chebyshev cardinal functions with unknown coefficients, at first. Then, the OM of V-O fractional derivative and some properties of the Chebyshev cardinal functions are employed to achieve a nonlinear algebraic equation corresponding to the performance index and a nonlinear system of algebraic equations corresponding to the dynamical system in terms of the unknown coefficients. Finally, the method of constrained extremum is applied, which consists of adjoining the constraint equations derived from the given dynamical system and the initial conditions to the performance index by a set of undetermined Lagrange multipliers. As a result, the necessary conditions of optimality are derived as a system of algebraic equations in the unknown coefficients of the state variable, control variable, and Lagrange multipliers. Furthermore, some numerical examples of different types are demonstrated with their approximate solutions for confirming the high accuracy and applicability of the proposed method.

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1. Introduction

During the last few decades, the subject of fractional calculus (derivatives and integrals of arbitrary order) has received the attention of many scientists in Mathematics, Physics and Engineering [1]. Many real-world physical systems display fractional order dynamics and their behavior is governed by fractional differential equations (FDEs) [2]. It is worth noting that the most important advantage of using FDEs is their nonlocal property [3], which means that the next state of a dynamical system depends not only on its current state but also on all of its previous states. Therefore, the memory effect of these derivatives is one of the main reasons to use them in various applications.

Since the order of fractional derivatives and integrals may take any arbitrary value, another extension is considering variable orders. This provides an extension of the classical fractional calculus, namely V-O fractional calculus. In fact, this subject is a generalization of fractional calculus where the order of fractional derivatives is known functions and depends on the time. These types of derivatives have been appeared and applied in many physical applications such as [4–6]. The authors of [7] have investigated the advantages of using V-O fractional derivatives rather than constant order fractional derivatives.

Optimal control theory is a branch of optimization theory focused on minimizing a cost or maximizing a payoff. Optimal control problems (OCPs) appear in engineering, science, economics, and many other fields. The fractional optimal control theory is a relatively new area in mathematics and engineering disciplines [8]. Fractional optimal control problems (FOCPs) can be defined using different definitions of fractional derivatives, like the Riemann–Liouville and Caputo fractional derivatives as the most important ones. FOCPs have gained much attention for their many applications in engineering and physics. For example, it has been illustrated that materials with memory and hereditary effects, and dynamical processes, including gas diffusion and heat conduction, in fractal porous media can be modeled by fractional order models better than the integer-order models [9,10]. An interested reader is referred to [11–18] for some recent studies on FOCPs.

Variable-order fractional optimal control problems (V-OFOCPs) can be defined with respect to different definitions of V-O fractional derivatives like the Riemann–Liouville and Caputo derivatives as the most important ones. Some recent works on the V-OFOCPs can be found in [19–21]. In general, most of the V-OFOCPs do not have exact analytic solutions, so approximate and numerical methods must be used.

There is a growing interest in using interpolate approximate base function to deal with various problems. The main characteristic of the approach using this technique is that it reduces these problems to those of solving a system of algebraic equations thus greatly simplifying the problem. In recent years, the cardinal functions have been finding an important role in the numerical analysis. Especially, valuable efforts have been spent, by researchers, on introducing novel ideas for the numerical solution of various functional equations by using the superior properties of these functions, e.g. [22–26]. In [22], the authors presented a numerical technique based on the Chebyshev cardinal functions for solving a class of parabolic partial differential equations with a time-dependent coefficient subject to an extra measurement. Their proposed method is derived by expanding the required approximate solution as the elements of the Chebyshev cardinal functions and employing the OM of derivative of these basis functions to reduce the problem to a set of algebraic equations. The authors of [23] proposed a numerical method based on the Chebyshev cardinal functions in conjunction with their OM derivative to the solution of a class of fourth-order integro-differential equations. In