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Brief paper

Synthesis of Petri net supervisors for FMS via redundant constraint elimination☆

Bo Huang[a], MengChu Zhou[b,c,1], GongXuan Zhang[a]

[a] School of Computer Science and Engineering, Nanjing University of Science and Technology, Nanjing 210094, China

[b] Department of Electrical and Computer Engineering, New Jersey Institute of Technology, Newark, NJ 07102, USA

[c] Key Laboratory of Embedded System and Service Computing, Ministry of Education, Tongji University, Shanghai 201804, China

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ABSTRACT

The Minimal number of Control Places Problem (MCPP), which is formulated to obtain optimal and structurally minimal supervisors, needs extensive computation. The current methods to reduce the computational burden have mainly focused on revision of the original formulation of MCPP. Instead, this paper presents methods to accelerate its solution by eliminating its redundant reachability constraints. The optimization problem scale required for supervisor synthesis is thus drastically reduced. First, a sufficient and necessary condition for a reachability constraint to be redundant is established in the form of an integer linear program (ILP), based on a newly proposed concept called feasible region of supervisors. Then, two kinds of redundancy elimination methods are proposed: an ILP one and a non-ILP one. Most of the redundant reachability constraints can be eliminated by our methods in a short time. The computational time to solve MCPP is greatly reduced after the elimination, especially for large-scale systems. The obtained supervisors are still optimal and structurally minimal. Finally, numerical tests are conducted to show the efficiency and effectiveness of the proposed methods.

1. Introduction

Various types of jobs concurrently compete for a limited number of resources such as numerically controlled machines, robots, buffers, sensors, and inspection stations in a flexible manufacturing system (FMS). Improper resource allocation may cause deadlocks, which may block and stall all activities in FMS.

Petri nets (PNs) play an important role in modeling and analyzing the behavior of FMS (Huang, Jiang, & Zhang, 2014; Huang, Shi, & Xu, 2012; Murata, 1989; Wu, 1999; Zhou & DiCesare, 1993; Zhou, DiCesare, & Desrochers, 1992; Zhou & Wu, 2010) and addressing the deadlock problems (Ezpeleta, Colom, & Martinez, 1995; Hu, Zhou, Li, & Tang, 2013; Li, Hu, & Wang, 2007; Li & Zhou, 2006). To prevent deadlocks in FMSs, we mainly have two kinds of PN-based analysis methods: structural analysis (Huang, Jeng, Xie, & Chung, 2001; Li & Zhou, 2006; Wang, Wang, & Yu, 2013; Wang, Wang, Yu, & Zhao, 2012b; Wang, Wang, Zhou, & Li, 2012a; Wang, Wu, & Yang, 2015; Xing, Zhou, Shi, & Ren, 2008) and reachability graph analysis (Chen & Li, 2012; Chen, Li, Khalgui, & Mosbahi, 2011; Ghaffari, Rezg, & Xie, 2003; Uzam & Zhou, 2006). The former often obtains a control policy by controlling special structures of a PN model, e.g., resource-transition circuits (Xing et al., 2008; Xing, Zhou, Wang, Liu, & Tian, 2011) and siphons (Huang et al., 2001; Li & Zhou, 2006; Liu, Li, & Zhou, 2013). The control law of this method is usually simple but the resultant model is not optimal in general. The latter can obtain a controlled model with optimal or nearly-optimal behavior. Note that optimality in this work means the maximal permissiveness in terms of reachable states excluding those deadlocks and states that inevitably evolve into deadlocks.

In Chen and Li (2011), a reachability graph-based method is proposed to obtain an optimal liveness-enforcing supervisor with the fewest control places for FMS modeled by PN. The reachability graph of the PN is divided into a live zone (LZ) and a deadlock zone (DZ). The markings in LZ are the legal ones of an FMS, and

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E-mail addresses: huangbo@njust.edu.cn (B. Huang), zhou@njit.edu (M. Zhou), gongxuan@njust.edu.cn (G. Zhang).

[1] Tel.: +1 973 5966282; fax: +1 973 5965680.

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those in DZ are deadlocks or will inevitably lead to deadlocks. First-met bad markings (FBMs) first proposed by Uzam and Zhou (2006) are those in DZ that are immediately reachable from some in LZ. A vector covering method (Chen et al., 2011) is used to reduce the number of markings to be considered in supervisor synthesis from the set of legal markings and the set of FBMs to a minimal covering set $\mathcal{M}_L^*$ of legal markings and a minimal covered set $\mathcal{M}_F^*$ of FBMs, respectively. An optimal supervisor with the fewest control places can be obtained by solving a Minimal number of Control Places Problem (MCPP) to forbid all markings in $\mathcal{M}_F^*$ and permit all markings in $\mathcal{M}_L^*$ when MCPP has an optimal solution. The supervisor obtained by the method is optimal and structurally minimal in terms of the number of control places. However, it has a limitation: the computational burden is extremely heavy, especially for large models.

Existing speedup techniques have mainly centered on revising the formulation of MCPP. Chen, Li, and Zhou (2012) propose an iterative method to design an optimal supervisor via place invariants (PIs). At each iteration, a Maximal number of Forbidding FBM Problem (MFFP) is solved to forbid as many FBMs as possible while permitting all legal markings. This method can reduce the computational time greatly, but it cannot guarantee the structural minimality of its derived supervisor. To find an optimal and structurally minimal supervisor quickly, Chen and Li (2012) propose a Minimal number of P-semiflows Problem (MPP) that has fewer constraints and variables than MCPP. However, MPP has an initially undecidable parameter, i.e., n_I, the number of PIs to be computed, and its efficiency greatly depends on the initial selection of n_I. Moreover, if n_I is set to be less than the unknown minimal number of control places of the problem, MPP fails to generate any solution.

Inevitably, an integer linear program (ILP) should be solved in MCPP. Yet, the methods able to accelerate MCPP solution by eliminating its redundant constraints have not been investigated. Our previous work (Huang, Zhu, Zhang, & Lu, 2015) proposed an inspiring method to eliminate redundant constraints of an ILP in supervisor synthesis. However, it is conducted in the context of designing an optimal PN supervisor with self-loops. This paper sheds new light on MCPP simplification by eliminating redundant reachability constraints. As a matter of fact, most of the constraints in MCPP are reachability ones, and many of them are redundant in supervisor synthesis. If they were eliminated, the scale of MCPP would be reduced, thereby speeding up the synthesis process.

In order to define a redundant reachability constraint, the concept of a feasible region of supervisors is introduced. It is defined as a set of all feasible combinations of control places for which all constraints in MCPP are satisfied. Based on this definition, a reachability constraint is said to be redundant if it can be eliminated without changing our concerned feasible region. A redundant constraint is inactive for all feasible supervisors. Therefore, its elimination does not change the solution to MCPP.

The rest of this paper is organized as follows. Section 2 gives the MCPP problem formulation. The definition of a redundant constraint and two kinds of redundant constraint elimination methods are proposed in Section 3. Section 4 provides experimental results. Finally, conclusions are given in Section 5.

2. Minimal number of control places problem

The structure optimization of an optimal supervisor is formulated as an MCPP (Chen & Li, 2011) with an objective function to minimize the number of control places in the supervisor. To aid supervisor synthesis, minimal covering set $\mathcal{M}_L^*$ of legal markings and minimal covered set $\mathcal{M}_F^*$ of FBMs are used. They can be calculated by a vector covering method, which can be found in Chen and Li (2011). Some notations are introduced as follows and MCPP is described in (1).

- a number of activity places in the PN model;
- b number of markings in $\mathcal{M}_F^*$;
- c number of markings in $\mathcal{M}_L^*$;
- D $(b \cdot c) \times a$ integer matrix of marking differences between $M_l \in \mathcal{M}_L^*$ and $M_j \in \mathcal{M}_F^*$, whose element is $d_{l,j}(p_i) = M_l(p_i) - M_j(p_i)$ with $i \in \{1, 2, \ldots, a\}$;
- $d_{l,j}$ a row of D with $l \in \{1, 2, \ldots, c\}$ and $j \in \{1, 2, \ldots, b\}$; an a integer vector of marking differences between $M_l \in \mathcal{M}_L^*$ and $M_j \in \mathcal{M}_F^*$;
- $D^{\langle *,j\rangle}$ a matrix obtained by including the rows related to $M_j \in \mathcal{M}_F^*$ in D;
- $\widetilde{D}^{\langle l,j\rangle}$ a matrix obtained by eliminating $d_{l,j}$ from D with $l \in \{1, 2, \ldots, c\}$ and $j \in \{1, 2, \ldots, b\}$;
- $\widetilde{D}^{\langle l,*,j\rangle}$ a matrix obtained by eliminating $d_{l,j}$ and the rows related to the found redundant constraints from $D^{\langle *,j\rangle}$;
- $\widetilde{D}_u^{\langle l,*,j\rangle}$ the uth row of $\widetilde{D}^{\langle l,*,j\rangle}$;
- E $(b\cdot(b-1))\times a$ integer matrix of marking differences between $M_k \in \mathcal{M}_F^*$ and $M_j \in \mathcal{M}_F^*$ $(k \neq j)$, whose element is $e_{k,j}(p_i) = M_k(p_i) - M_j(p_i)$ with $i \in \{1, 2, \ldots, a\}$;
- $e_{k,j}$ a row of E with $k, j \in \{1, 2, \ldots, b\}$ and $k \neq j$; an a vector of marking differences between M_k and M_j in $\mathcal{M}_F^*$;
- $f_{j,k}$ a binary variable: $f_{j,k} = 1$ if $M_k \in \mathcal{M}_F^*$ is forbidden by the PI related to $M_j \in \mathcal{M}_F^*$, otherwise $f_{j,k} = 0$;
- g_j an a nonnegative integer vector of the coefficients of the PI corresponding to $M_j \in \mathcal{M}_F^*$;
- h_j a binary variable: $h_j = 1$ if the PI related to $M_j \in \mathcal{M}_F^*$ is selected to compute a control place, otherwise $h_j = 0$;
- Γ a positive integer that should be chosen big enough;
- $\gamma_{l,j}$ a reachability constraint: $d_{l,j} \cdot g_j^T \leqslant -1$ with $l \in \{1, 2, \ldots, c\}$ and $j \in \{1, 2, \ldots, b\}$;
- $S(D, E)$ the system defined by constraints in MCPP;
- $\Omega(D, E)$ the feasible region of supervisors in $S(D, E)$.

$$
\begin{aligned}
&\min \sum_{j\in\{1,2,\ldots,b\}} h_j \\
&\text{subject to} \\
&d_{l,j} \cdot g_j^T \leqslant -1, \quad \forall l \in \{1, 2, \ldots, c\} \text{ and } \forall j \in \{1, 2, \ldots, b\} \\
&e_{k,j} \cdot g_j^T \geqslant \Gamma \cdot (f_{j,k} - 1), \quad \forall j,\ k \in \{1, 2, \ldots, b\} \text{ and } j \neq k \\
&f_{j,k} \leqslant h_j, \quad \forall j, k \in \{1, 2, \ldots, b\} \text{ and } j \neq k \\
&h_j + \sum_{k\in\{1,2,\ldots,b\},\ k\neq j} f_{k,j} \geqslant 1, \quad \forall j \in \{1, 2, \ldots, b\}.
\end{aligned} \tag{1}
$$

At the first sight, the above formulation may seem different from the one in Chen and Li (2011). However, they are in essence the same. In this problem, the reachability constraints are expressed by the inequalities containing vector $d_{l,j}$.

3. Eliminating redundant constraints

First, the definition of a redundant reachability constraint is given. If most of the reachability constraints are redundant and eliminated efficiently, the calculation of MCPP and the whole supervisor synthesis may be considerably facilitated. Then, two kinds of elimination methods are proposed. Finally, a method to integrate constraint elimination steps into the supervisor synthesis is given.

3.1. Definition of a redundant reachability constraint

The constraints in (1) determine the feasible region of optimal PN supervisors. For generalization, remove the objective function