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Orbit consensus of quantum networks based on interaction design[☆]

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Abstract

For a general quantum network system with a non-zero Hamiltonian H composed of n identical m-level quantum subsystems, any symmetric consensus state in the interaction picture exactly corresponds to an orbit in the Schrödinger picture, which is called the H-orbit of the symmetric consensus state. By using the interaction picture transformation and the tool of the LaSalle invariance principle, this paper analyzes the orbit consensus of this quantum network and designs the corresponding swapping operators such that the system converges to the H-orbit of the target symmetric consensus state that exists in the interaction picture. In particular, we prove the convergence of the quantum network to the H-orbit when the quantum interaction graph is connected and the system Hamiltonian is permutation invariant. The orbit consensuses of a four-qubit network system and a quantum network of three identical three-level subsystems are achieved numerically, which verifies the correctness of our theoretical results and the effectiveness of the designed swapping operators.

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1. Introduction

The consensus problem is one of fundamental problems in multi-agent systems. It means that all individuals in the network tend to the same state via the interactions between each individual and its neighbors. Due to important applications in the fields of distributed control and distributed computation, classical consensus problems have been extensively studied [1–6]. With the development of quantum information technology in recent years [7], research on quantum control theory and methods has been gaining more and more fruits [8–18]. The consensus problem of quantum networks has also been attracting wide attention [19–24], for instance, it plays an important role for distributed quantum computation and quantum communication.

In distributed quantum computation, different quantum systems are usually regarded as quantum nodes to form a quantum network. For instance, a network of quantum bits as nodes is called a qubit network, where each qubit is a two-state quantum system that can be realized by an electron or a photon in quantum informatics. Different from the classical consensus, some non-classical correlations may exist between quantum nodes. Therefore, the consensus of a quantum network cannot be simply thought of as that all nodes in the network are in the same state. Based on the permutation invariance of consensus states, Mazzarella et al. [20] proposed the concepts of four classes of consensus states for quantum networks (i.e., σ-expectation consensus, reduced state consensus, symmetric state consensus (SSC), single σ-measurement consensus), and analyzed the relationships between them. These four classes of consensus states have also been involved in [21–30].

The evolution of quantum network systems can be divided into two classes: discrete-time case and continuous-time case. In [20], a gossip algorithm for discrete-time quantum networks was first proposed and the symmetric state consensus was achieved via discrete-time swapping operations between two neighbor quantum subsystems. Ticozzi [21] further gave an improved quantum gossip algorithm and obtained high-purity consensus states, and at the same time achieved σ-measurement consensus. A quantum consensus algorithm with distributed feedback control was proposed in [22], where the convergence of the network system to the symmetric consensus state was analyzed in the framework of stochastic Lyapunov stability theory. Also, the algorithm was used to prepare W-entangled states. For the consensus problem of quantum networks with continuous-time dynamics, Ticozzi et al. [23] achieved the convergence of the network to a state set that is invariant with respect to subsystem permutations by transforming the continuous-time quantum consensus problem into a symmetry problem in the group theory and employing the tool of continuous-time quantum dynamical semigroup generators and the Lyapunov theory. Shi et al. [24,25] used a continuous-time Markovian master equation to describe the evolution of the network state, where the information exchange between two neighboring subsystems was represented by a continuous-time swapping operator; and built the connections between quantum consensus and classical consensus via the graph-theoretic methods. Further, they provided several convergence results for quantum networks with time-invariant quantum interaction graphs, as well as necessary and sufficient conditions for exponential consensus and asymptotic consensus of quantum networks with time-varying quantum interaction graphs. Ref. [26] established several convergence results for the consensus of continuous-time quantum networks with directed interaction graphs via the Perron–Frobenius theorem. Jafarizadeh [27,28] achieved the optimization of the convergence rate of continuous/discrete-time quantum consensus by transforming quantum consensus problems into classical consensus optimization problems. It should be pointed out

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