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Brief paper

Decentralized filtered dynamic inversion for uncertain minimum-phase systems☆

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ABSTRACT

Decentralized filtered dynamic inversion is a control method for uncertain linear time-invariant systems that are minimum phase, have multi-input multi-output decentralized subsystems, and are potentially subject to unknown disturbances. This controller requires limited model information, specifically, knowledge of the relative degree and an estimate of the first nonzero Markov parameter for each local subsystem. Decentralized filtered dynamic inversion is effective for command following and rejection of unknown disturbances. We derive the decentralized filtered-dynamic-inversion controller and analyzes the closed-loop stability and performance (i.e., command-following error). We show that for sufficiently large choice of a single control parameter the closed-loop system is asymptotically stable, and the average power of the performance is arbitrarily small.

1. Introduction

Decentralized control systems are composed of interconnected subsystems, which have local sensors and local controllers. The defining characteristic of decentralized control is that each local controller has access to only local sensor measurements. Time-invariant control approaches can be used for systems without decentralized fixed modes (Corfmat & Morse, 1976; Davison, 1976; Ünyelioğlu & Özgüler, 1992; Wang & Davison, 1973), and time-varying approaches have been proposed for systems with decentralized fixed modes (Anderson & Moore, 1981; Khargonekar & Özgüler, 1994; Wang, 1982; Willems, 1989). However, Anderson and Moore (1981), Corfmat and Morse (1976), Davison (1976), Khargonekar and Özgüler (1994), Ünyelioğlu and Özgüler (1992), Wang (1982), Wang and Davison (1973) and Willems (1989) require complete and accurate model information.

Adaptive control methods have been developed for decentralized systems with incomplete or inaccurate model information (Gavel, 1989; Ioannou, 1986; Ioannou & Kokotovic, 1985; Narendra & Oleng, 2002; Narendra, Oleng, & Mukhopadhyay, 2006; Pagilla, Dwivedula, & Siraskar, 2007; Polston & Hoagg, 2015; Shi & Singh, 1992; Wen & Soh, 1999). In particular, Gavel (1989) and Shi and Singh (1992) present decentralized adaptive controllers that rely on local full-state feedback for stabilization. These local full-state feedback adaptive techniques are extended in Narendra and Oleng (2002), Narendra et al. (2006), Pagilla et al. (2007) and Polston and Hoagg (2015) to address command following. However, Narendra and Oleng (2002), Narendra et al. (2006) and Pagilla et al. (2007) require centralized reference models and centralized commands. Decentralized output-feedback adaptive controllers are presented in Ioannou (1986), Ioannou and Kokotovic (1985) and Wen and Soh (1999) for stabilization of linear time-invariant (LTI) systems, where each local single-input single-output (SISO) subsystem is minimum phase. These methods guarantee that the command-following error is bounded but do not generally make the magnitude of the error small.

In sum, the adaptive controllers of Gavel (1989), Ioannou (1986), Ioannou and Kokotovic (1985), Narendra and Oleng (2002), Narendra et al. (2006), Pagilla et al. (2007), Polston and Hoagg (2015), Shi and Singh (1992) and Wen and Soh (1999) are effective for stabilization and, to some extent, address command following; however, none of these techniques address rejection of unknown disturbances. Furthermore, all of these techniques are restricted to local subsystems that are either SISO or have local full-state feedback.

We present a decentralized controller that addresses stabilization and command following for uncertain LTI systems that have multi-input multi-output (MIMO) subsystems and are potentially subject to unknown disturbances, which need not be deterministic. A key concept used in this paper is *centralized dynamic inversion*,

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which makes invariant zeros of the open loop into eigenvalues of the closed loop. Dynamic inversion is referred to as feedback linearization for nonlinear systems.

Dynamic inversion has important limitations, namely, the full state is required for feedback, the plant model must be known, and all exogenous disturbances must be measured for disturbance rejection. To overcome these limitations, Hoagg and Seigler (2013a) presents a centralized control, termed *filtered dynamic inversion*. This approach combines standard dynamic inversion with a linear filter to obtain a control, which uses only output feedback, requires limited model information (specifically, the relative degree and the first nonzero Markov parameter), and does not require measurement of the disturbance. The centralized controller of Hoagg and Seigler (2013a) is effective for stabilization, command following, and disturbance rejection for uncertain MIMO LTI systems that are minimum phase (i.e., the invariant zeros are contained in the open-left-half complex plane). Moreover, Hoagg and Seigler (2013a) shows that filter dynamic inversion makes the average power of the command-following error arbitrarily small. The results of Hoagg and Seigler (2013a) are extended to address centralized nonlinear systems in Hoagg and Seigler (2013b).

We present a new decentralized controller, termed *decentralized filtered dynamic inversion* (D-FDI), which is effective for uncertain LTI systems that are minimum phase, have MIMO subsystems, and are potentially subject to unknown disturbances. We combine centralized dynamic inversion with a linear filter, which can be interpreted as a low-pass filter whose cutoff frequency depends on a single parameter. The introduction of the linear filter removes nonlocal input–output channels from the centralized dynamic-inversion controller, yielding a decentralized control that requires limited model information, specifically, knowledge of the relative degree and an estimate of the first nonzero Markov parameter for each local subsystem. We show that for a sufficiently large parameter, the performance (i.e., command-following error) with D-FDI approximates to arbitrary accuracy the performance with centralized dynamic inversion, which cannot be implemented because it relies on full-state feedback, unknown plant parameters, and unmeasured disturbances. This paper shows that for sufficiently large parameter the closed-loop system is asymptotically stable, and the average power of the performance is arbitrarily small. This paper adopts techniques from Hoagg and Seigler (2013a) but goes beyond (Hoagg & Seigler, 2013a) by addressing decentralized control.

2. Problem formulation

Consider the multi-input LTI system

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{\ell} B_i u_i(t) + w(t), \tag{1}$$

where $t \geq 0$, $x(0) \in \mathbb{R}^n$ is the initial condition, $x(t) \in \mathbb{R}^n$ is the state, $w(t) \in \mathbb{R}^n$ is an unknown-and-unmeasured disturbance, and, for all $i \in \mathcal{I} \triangleq \{1, 2, \ldots, \ell\}$, $u_i(t) \in \mathbb{R}^{m_i}$ is a control. For all $i \in \mathcal{I}$, consider the measurements

$$y_i(t) = C_i x(t), \tag{2}$$

where $C_i \in \mathbb{R}^{m_i \times n}$. We consider the decentralized control problem, where for each $i \in \mathcal{I}$, the *local control* u_i can depend on the *local measurement* y_i but cannot depend on the *nonlocal measurements* $\{y_j\}_{j\in\mathcal{I}\setminus\{i\}}$.

Next, define $B \triangleq [B_1 \cdots B_\ell] \in \mathbb{R}^{n\times m}$ and $C \triangleq [C_1^{\mathrm{T}} \cdots C_\ell^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{m\times n}$, where $m \triangleq m_1 + \cdots + m_\ell$. We assume that (A, B, C) is controllable and observable. Define $y \triangleq [y_1^{\mathrm{T}} \cdots y_\ell^{\mathrm{T}}]^{\mathrm{T}}$ and $u \triangleq [u_1^{\mathrm{T}} \cdots u_\ell^{\mathrm{T}}]^{\mathrm{T}}$. For all $i \in \mathcal{I}$, the *relative degree* d_i from u to y_i is the smallest integer k such that $C_i A^{k-1} B$ is nonzero. The disturbance w is d-times differentiable, and $w, \dot{w}, \ldots, w^{(d)}$ are bounded, where $d \triangleq \max\{d_1, d_2, \ldots, d_\ell\}$. For all $i, j \in \mathcal{I}$, define $H_{i,j} \triangleq C_i A^{d_i-1} B_j$, and define

$$H \triangleq \begin{bmatrix} H_{1,1} & \cdots & H_{1,\ell} \\ \vdots & \ddots & \vdots \\ H_{\ell,1} & \cdots & H_{\ell,\ell} \end{bmatrix} \in \mathbb{R}^{m\times m}. \tag{3}$$

We make the following assumptions:

(A1) If $\lambda \in \mathbb{C}$ and $\det \begin{bmatrix} \lambda I - A & B \\ C & 0 \end{bmatrix} = 0$, then $\operatorname{Re} \lambda < 0$.

(A2) $d_1, d_2, \ldots, d_\ell$ are known.

(A3) $H_{1,1}, H_{2,2}, \ldots, H_{\ell,\ell}$, and H are nonsingular.

(A4) For all $i \in \mathcal{I}$, there exists a known $\bar{H}_i \in \mathbb{R}^{m_i\times m_i}$ such that $\bar{H}_i H_{i,i}^{\mathrm{T}} - H_{i,i} H_{i,i}^{\mathrm{T}}$ is positive semidefinite.

Next, define

$$M \triangleq \operatorname{diag}(H_{1,1}, \ldots, H_{\ell,\ell}) \in \mathbb{R}^{m\times m}, \tag{4}$$

$$\bar{M} \triangleq \operatorname{diag}(\bar{H}_1, \ldots, \bar{H}_\ell) \in \mathbb{R}^{m\times m}, \tag{5}$$

where diag $(\cdot)$ is a block-diagonal matrix whose block-diagonal elements are given by the arguments of the operator. We make the following additional assumption:

(A5) If $H \neq M$, then there exist known $\nu_1 > 0$ and $\nu_2 > 0$ such that if $\lambda \in \mathbb{C}$ and $\det(\lambda \bar{M} + H) = 0$, then $|\lambda| \leq \nu_1$ and $\operatorname{Re} \lambda \geq \nu_2$.

The plant (A, B, C) is otherwise unknown, and w and x are unmeasured. Assumption (A1) implies that (A, B, C) is minimum phase, that is, the invariant zeros are contained in the open-left-half complex plane. Assumption (A1) is required for centralized dynamic inversion and is also required for D-FDI.

Assumption (A3) states that H is nonsingular, which is also required in centralized dynamic inversion. Assumption (A3) also states that the first nonzero local Markov parameters $H_{i,i}$ are nonsingular. Sensor and actuator placement can often be used to ensure that (A3) is satisfied. For example, if the local control u_i is colocated with the local measurement y_i but not colocated with the nonlocal measurements $\{y_j\}_{j\in\mathcal{I}\setminus\{i\}}$, then (A3) is often satisfied. In fact, structural systems with colocated local sensors and actuators are minimum phase and have nonsingular $H_{1,1}, \ldots, H_{\ell,\ell}$ (Hoagg, Chandrasekar, & Bernstein, 2007; Lin & Juang, 1995). Section 6 considers examples that satisfy (A1)–(A5).

The parameter $\bar{H}_i$, which appears in (A4), can be interpreted as an estimate of $H_{i,i}$. If a local subsystem is SISO (i.e., $m_i = 1$), then (A4) requires that the sign of $H_{i,i}$ and an upper bound on the magnitude of $H_{i,i}$ are known. In this case, $\bar{H}_i$ satisfies (A4) if and only if $\operatorname{sgn}(\bar{H}_i) = \operatorname{sgn}(H_{i,i})$ and $|\bar{H}_i| \geq |H_{i,i}|$.

The nonlocal Markov parameters $H_{i,j}$, where $i \neq j$, are not assumed to be known. However, if $H \neq M$, then (A5) implies that the eigenvalues of $\bar{M}^{-1}H$ are contained in the open-right-half complex plane, an upper bound on the magnitudes of the eigenvalues of $\bar{M}^{-1}H$ is known, and a lower bound on the real parts of the eigenvalues of $\bar{M}^{-1}H$ is known. For many applications, sensor and actuator placement can be used to accomplish $H = M$. In this case, (A5) is not required.

Next, for all $i \in \mathcal{I}$, consider the polynomial matrices $\alpha_{\mathrm{m},i}(s) = s^{d_i}\alpha_{i,d_i} + \cdots + s\alpha_{i,1} + \alpha_{i,0}$ and $\beta_{\mathrm{m},i}(s) = s^{d_i}\beta_{i,d_i} + \cdots + s\beta_{i,1} + \beta_{i,0}$, where $\alpha_{i,d_i} = I_{m_i}$; $\alpha_{i,0}, \ldots, \alpha_{i,d_i-1} \in \mathbb{R}^{m_i\times m_i}$; $\beta_{i,0}, \ldots, \beta_{i,d_i} \in \mathbb{R}^{m_i\times m_i}$; and if $\lambda \in \mathbb{C}$ and $\det \alpha_{\mathrm{m},i}(\lambda) = 0$, then $\operatorname{Re} \lambda < 0$. For all $i \in \mathcal{I}$, define $p_i \triangleq \deg \beta_{\mathrm{m},i}(s) + 1$, and let $\mathbf{p} = \mathrm{d}/\mathrm{d}t$ denote the differential operator. For all $i \in \mathcal{I}$, consider the reference model $\alpha_{\mathrm{m},i}(\mathbf{p}) y_{\mathrm{m},i}(t) = \beta_{\mathrm{m},i}(\mathbf{p}) r_i(t)$, where $t \geq 0$; $r_i(t) \in \mathbb{R}^{m_i}$ is the reference-model command, which is p_i-times differentiable, and where $r_i, \dot{r}_i, \ldots, r_i^{(p_i)}$ are bounded; $y_{\mathrm{m},i}(t) \in \mathbb{R}^m$ is the reference-model output; and the initial condition is $y_{\mathrm{m},i}(0), \ldots, y_{\mathrm{m},i}^{(d_i-1)}(0)$