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Sampled-data controllability and stabilizability of Boolean control networks: Nonuniform sampling[☆]

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Abstract

Derived from a simplified intelligent traffic control system, sampled-data controllability and stabilizability of Boolean control networks are considered. Compared with the existing case of uniform (periodic) sampling in Boolean control networks, the nonuniform one is more general. Using linear span with integral coefficients, the distribution of sampling points can be obtained. Then by constructing novel systems, some necessary and sufficient conditions are proposed to determine sampled-data controllability and stabilizability. Finally, two illustrative examples, which are on apoptosis networks and traffic control systems, respectively, are worked out to show the effectiveness of the obtained results.

1. Introduction

To research genetic regulation, Kauffman firstly established the model of Boolean networks to describe behaviors of genes [1]. The variables in Boolean networks take only two values, 1 and 0, which represent high and low concentrations, genes being transcribed and quiescent, and so on, respectively. These variables are evolutionary under a series of Boolean functions, which are combinations of logical operators, such as, conjunction (AND), disjunction (OR) and negation (NOT). Numerous results reveal that Boolean networks are capable of exhibiting

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dynamics of large-scale biological networks and effective in simulating, even predicting the behaviours of networks, although they are quite simplified in form [2–5].

To depict external inputs and considerable uncertainties, Boolean networks are generalized to Boolean control networks, probabilistic Boolean networks [6,7], etc. Recently, combined with a new mathematical tool, semi-tensor product of matrices, the relevant works on Boolean (control) networks have been flourishing. Under semi-tensor product, traditional Boolean functions can be rewritten as equivalent algebraic representations [8], which is convenient to analyze control problems of Boolean control networks, including controllability [9–11] and observability [12,13], stabilizability [14–18] and optimal control [19–21], synchronization [22,23] and decomposition [24,25]. Moreover, semi-tensor product has been applied to some practical issues, such as, game theory [26,27], digital circuits [28], internal combustion engine [29].

In this paper, we consider two fundamental and correlative issues of Boolean control networks, controllability and stabilizability [15,19], both of which can be regarded as the reachability to some extent. The former one concerns the strong connectivity or the reachability relation between arbitrary two states, and the latter pays more attention to the weak connectivity or whether the system can keep on a state which is (globally) reachable.

Via semi-tensor product, controllability and stabilizability of Boolean control networks were investigated in [9] and [14] for the first time, respectively. Then the state space approach adopted in [9] was generalized to input-state spaces [10], under which suitable control signals can be designed conveniently. After that, arising from practical issues, the controllability of some more specific Boolean control networks was studied, for example, ones with delays in states [30] or with forbidden states [11]. In [15,19], the intimate connection between controllability and stabilizability was given. In addition, [19] proved that a Boolean control network being stabilizable to a given state is equivalent to it being stabilizable to this state under state feedback. And [15] provided a constructive method to determine all minimum-time state-feedback stabilizers. Subsequently, the types of stabilizer were extended further. Time invariant and variant output-feedback stabilizers were proposed and designed [16,17]. Until now, the algorithms to find all the time invariant output-feedback and state-feedback stabilizers have been established [17,31].

In modern control theory, sampling transforms continuous signals into discrete ones, which was extensively used in neural networks [32], multi-agent systems [33–36] and so on. A band-limited signal can be restored from samples if the (average) sampling rate meets some conditions. The errors caused by instruments are technically inevitable, thus nonuniform-sampling is more accurate and effective. In this article, the networks that we aim to explore are not general Boolean control networks but ones under nonuniform-sampled inputs, which can be regarded as the generalization of (uniformly) sampled-data control of Boolean control networks investigated in [37]. Since nonuniform-sampling reduces to uniform-sampling when sampling points are taken equally spaced in time. Namely, periods of nonuniform-sampling are more flexible. In Section 3, we obtain a model with nonuniform sampling periods, which is the source of nonuniform-sampling issues in Boolean control networks. To the best of our knowledge, there is no available result about Boolean control networks under nonuniform-sampled inputs.

The reminder of this study is structured as follows. Section 2 provides preliminaries on notations and semi-tensor product. By a simplified intelligent traffic control system, Boolean control networks under nonuniform-sampled inputs are introduced in Section 3. Section 4 contains our main results, sampled-data stabilizability and controllability. Two illustrative