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Brief paper

Further deleterious effects of the dissipation obstacle in control-by-interconnection of port-Hamiltonian systems☆

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ABSTRACT

The presence of dissipation hampers our ability to shape the energy of port-Hamiltonian systems using control-by-interconnection methods—a phenomenon called the dissipation obstacle. In particular, the Casimir functions that are used to shift the energy function cannot depend on the coordinates where dissipation is present if we use passive controllers. Recently, it was proposed to relax the latter condition using non-passive controllers that inject energy into the system to be able to create the required Casimir functions. In this note we prove that, alas, even if the Casimirs can be created with active controllers the dissipation obstacle stymies the possibility to assign an energy function with the minimum at an equilibrium point. As a corollary we prove that the deleterious effect of pervasive dissipation does not only stem from the inability of the controller to inject the (infinite) energy required for stabilization—as it was stated in earlier publications.

1. Introduction

In this paper we are interested in the problem of stabilization of port-Hamiltonian (pH) systems via the passivity-based control (PBC) technique of Control-by-Interconnection (CbI), first proposed in Dalsmo and van der Schaft (1999) and extensively studied in Ortega, van der Schaft, Mareels, and Maschke (2001) and Ortega, Castaños, van der Schaft, and Astolfi (2008). In CbI the controller is another pH system with its own state variables and energy function, which is interconnected in a power-preserving way to the plant. The overall system is still pH with new energy function the sum of the energy functions of the plant and the controller. To assign to the overall energy function a desired shape[1] it is necessary to "relate" the states of the plant and the controller via the generation of invariant manifolds—defined by Casimir functions. In its basic formulation, called Standard CbI, only the plant output is measurable and considers the classical output feedback interconnection. In this case, the Casimir functions are fully determined by the plant, which imposes a severe restriction on the plant dissipation structure. It has been shown in Ortega et al. (2001) that, roughly speaking, "dissipation cannot be present on the coordinates to be shaped". This, so-called, *dissipation obstacle* stymies the use of Standard CbI for applications other than mechanical systems where the coordinates to be shaped are typically positions, which are unaffected by friction.

It is well-known (Ortega et al., 2001) that the origin of the dissipation obstacle is the existence of *pervasive* dissipation, *i.e.*, dissipation that is present even at the equilibrium state, requiring for stabilization to extract an infinite amount of energy from the controller. Since the controller in Standard CbI is a passive system and the energy that can be extracted from a passive system is bounded (van der Schaft, 1999; van der Schaft & Jeltsema, 2014) it

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[1] In the context of equilibrium stabilization by "shaping the energy function" we mean assign to it a minimum at the desired equilibrium.

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is clear that Standard CbI will not be applicable for this class of systems. To fulfill the (infinite) energy demand of systems with pervasive dissipation it was recently proposed in Koopman and Jeltsema (2012) to use in CbI *active*, as opposed to passive, controllers—that is, controllers with sources that can provide an infinite amount of energy. The main objective of this paper is to prove that, even if the Casimirs can be created with active controllers, the dissipation obstacle stymies the possibility to assign an energy function with the minimum at an equilibrium point.

As shown in Ortega et al. (2008) the dissipation obstacle can be overcome exploiting alternative pH representations of the system that generate new port variables and applying CbI through these port variables. See also Ortega and Borja (2014) and Venkataraman and van der Schaft (2009) for some more recent developments. This modified CbI controllers, which require additional state measurements, are not investigated here.

The remaining of the paper is organized as follows. In Section 2 we give some background material on Standard CbI for pH systems, Casimir functions and the dissipation obstacle. Section 3 contains our main result and Section 4 gives an illustrative example. We wrap-up the paper with some concluding remarks in Section 5.

Notation. For $x \in \mathbb{R}^n$, $|x|$ is the Euclidean norm. All the functions and mappings in the paper are assumed sufficiently smooth. For mappings of scalar argument $g : \mathbb{R} \to \mathbb{R}^s$, g' and g'' denote their first and their second order derivative, respectively. For functions $H : \mathbb{R}^n \to \mathbb{R}$ we define the operators $\nabla H := (\frac{\partial H}{\partial x})^\top$ and $\nabla^2 H := \frac{\partial^2 H}{\partial x^2}$. Also, for mappings $W : \mathbb{R}^n \times \mathbb{R}^{n_c} \to \mathbb{R}$ the operators $\nabla_x W(x, x_c) := (\frac{\partial W}{\partial x})^\top$ and $\nabla_{x_c} W(x, x_c) := (\frac{\partial W}{\partial x_c})^\top$ are defined. For vector functions $\mathcal{C} : \mathbb{R}^n \to \mathbb{R}^m$, we define its (transposed) Jacobian matrix $\nabla \mathcal{C}(x) = [\nabla \mathcal{C}_1(x), \ldots, \nabla \mathcal{C}_m(x)]$. For the distinguished element $x_\star \in \mathbb{R}^n$ and any mapping $F : \mathbb{R}^n \to \mathbb{R}^s$ we denote $F_\star := F(x_\star)$.

2. Background material

In this section, we briefly review the CbI method applied to the stabilization of pH systems described in input-state-output form (van der Schaft, 1999; van der Schaft & Jeltsema, 2014), recall the role of the dissipation obstacle (Ortega et al., 2008) and the recent proposal of Koopman and Jeltsema (2012) to use active (as opposed to passive) controllers to generate the Casimir functions needed to complete the controller design.

2.1. PH systems and the dissipation obstacle

The input-state-output representation of pH systems is of the form van der Schaft (1999)

$$\Sigma_{(u,y)} : \begin{cases} \dot{x} &= [J(x) - R(x)]\nabla H(x) + g(x)u, \\ y &= g^\top(x)\nabla H(x), \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u, y \in \mathbb{R}^m$, $m \le n$, are conjugated variables whose product has units of power, $H : \mathbb{R}^n \to \mathbb{R}$ is the systems stored energy, $J, R : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, with $J(x) = -J^\top(x)$ and $R(x) = R^\top(x) \ge 0$, are the interconnection and damping matrices, respectively, and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ is the input matrix, which is full rank. To simplify the notation in the sequel, we define the matrix $F : \mathbb{R}^n \to \mathbb{R}^{n\times n}$

$$F(x) := J(x) - R(x).$$

The power-balance equation of the system to be controlled is

$$\dot{H} = -[\nabla H(x)]^\top R(x)\nabla H(x) + u^\top y. \tag{2}$$

This power balance reveals the following feature that is going to be important for our subsequent developments. Let us assume that the control objective is to drive the systems state towards a given equilibrium point $x_\star \in \mathbb{R}^n$. Since $\dot{H}$ evaluated at $x_\star$ should be equal to zero and $R \ge 0$ it follows that the power dissipated at the equilibrium should be compensated by the controller. That is,

$$u_\star^\top y_\star = \nabla H_\star^\top R_\star \nabla H_\star. \tag{3}$$

On the other hand, it is known (van der Schaft, 1999) that the power that can be extracted from a passive system is bounded. Consequently, if the system continues dissipating power at the equilibrium[2] – situation that is known as *pervasive dissipation* – it cannot be stabilized with a passive controller. That is,

$$R_\star \nabla H_\star = 0, \tag{4}$$

is a *necessary* condition for stabilization with passive controllers. A system that *does not* verify condition (4) is said to be *constrained by the dissipation obstacle*. This concept was first introduced in Ortega et al. (2001) and its implications are studied in detail in Castaños and Ortega (2009) and Ortega et al. (2008). One of the objectives of this paper is to contribute to our better understanding of this restriction.

It is interesting to note that the identity (3) can be established without invoking the power balance argument. Indeed, since at equilibrium

$$(J_\star - R_\star)\nabla H_\star + g_\star u_\star = 0, \qquad y_\star = g_\star^\top \nabla H_\star,$$

we have that

$$\begin{aligned} u_\star^\top y_\star &= u_\star^\top g_\star^\top \nabla H_\star \\ &= -[(J_\star - R_\star)\nabla H_\star]^\top \nabla H_\star \\ &= -\nabla H_\star^\top (J_\star - R_\star)\nabla H_\star \\ &= \nabla H_\star^\top R_\star \nabla H_\star. \end{aligned}$$

It is important, however, to clarify that – except for the case when $n - m = 1$, when they coincide – the set of assignable equilibria of the pH system (1) is a *strict subset* of the set where the power is balanced. More precisely, the sets

$$\mathcal{E} := \{x \in \mathbb{R}^n \mid g^\perp(x)F(x)\nabla H(x) = 0\},$$
$$\mathcal{P} := \{x \in \mathbb{R}^n \mid \dot{H}(x) = 0\},$$

verify $\mathcal{E} \subseteq \mathcal{P}$, where $g^\perp : \mathbb{R}^n \to \mathbb{R}^{(n-m)\times n}$ is a full rank left annihilator of $g(x)$, that is, $g^\perp(x)g(x) = 0$ and rank $\{g^\perp(x)\} = n - m$. See Sanchez, Ortega, Griños, Bergna, and Molinas-Cabrera (2014) for a discussion on this point.

2.2. Standard Control-by-Interconnection

In CbI (Dalsmo, 2009; Ortega et al., 2008; van der Schaft, 1999), the controller is another pH system

$$\Sigma_{(u_c,y_c)} : \begin{cases} \dot{x}_c &= F_c(x_c)\nabla H_c(x_c) + g_c(x_c)u_c, \\ y_c &= g_c^\top(x_c)\nabla H_c(x_c), \end{cases} \tag{5}$$

with state[3] $x_c \in \mathbb{R}^{n_c}$, $u_c, y_c \in \mathbb{R}^m$, $m \le n_c$, $H_c : \mathbb{R}^{n_c} \to \mathbb{R}$, and $F_c : \mathbb{R}^{n_c} \to \mathbb{R}^{n_c\times n_c}$. As before,

$$F_c(x_c) := J_c(x_c) - R_c(x_c),$$

with $J_c(x_c) = -J_c^\top(x_c)$ and $R_c(x_c) = R_c^\top(x_c)$. It is important to underscore that, following the suggestion of Koopman and Jeltsema (2012), we *do not* assume $R_c(x_c)$ to be positive semidefinite.

[2] For instance, if the steady-state value of the current flowing through a resistor is nonzero.

[3] To simplify the presentation we have taken the dimensions of the input and output spaces of the controller equal to the ones of the plant, *i.e.*, m.