Automatica 61 (2015) 302–307

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

An iterative partition-based moving horizon estimator with coupled inequality constraints☆

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ARTICLE INFO

Article history:
Received 16 April 2014
Received in revised form
8 June 2015
Accepted 12 August 2015
Available online 5 September 2015

Keywords:
Moving horizon estimation
System partitioning
Sensitivity-driven optimization
Iterative algorithm
Stability

ABSTRACT

We propose an iterative, partition-based moving horizon state estimator for large-scale linear systems that consist of interacting subsystems. Every subsystem estimates its own state and disturbance variables, taking into account the estimates received from neighboring subsystems. Compared to other partition-based moving horizon estimators, the proposed method has two unique features: it can handle coupled inequality constraints on the estimated variables and its state estimates come arbitrarily close to the optimal state estimates of a centralized moving horizon estimator. The applicability and performance of the proposed method are demonstrated on a numerical example and convergence and asymptotic stability are rigorously proven.

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1. Introduction

Especially for large-scale systems composed of interconnected subsystems, partition-based state estimation has been recognized as a promising alternative to decentralized and centralized approaches. In partition-based estimation, the state of each subsystem is estimated individually by dedicated subsystem estimators, which exchange information in order to compensate for their limited knowledge about the other subsystems and their measurements. In this way, partition-based estimators aim to combine the intuitive structure of decentralized estimation with the superior performance of centralized estimation.

In particular, this paper focuses on partition-based moving horizon estimation (PMHE). Recent approaches are reviewed in Christofides, Scattolini, Muñoz de la Peña and Liu (2013), including most notably a number of non-iterative methods with guaranteed stability (Farina, Ferrari-Trecate, & Scattolini, 2010). An iterative alternative method was proposed in Schneider, Scheu, and Marquardt (2013). It is called *sensitivity-driven*, partition-based moving horizon estimator (S-PMHE), since the sensitivities of neighboring subsystems are taken into account by the local estimators. S-PMHE has been successfully applied to a nonlinear case study from chemical engineering (Schneider, Scheu, & Marquardt, 2014). The unique feature of this method is that, iteratively, its state estimates approach the optimal state estimates of the corresponding centralized moving horizon estimator (CMHE) as proposed in Rao, Rawlings, and Lee (2001).

The main contribution of this paper is an extension of S-PMHE (Schneider et al., 2013) that enables *coupled inequality constraints* on the estimated state and disturbance variables. This extension adds another unique feature to S-PMHE, as no other partition-based moving horizon estimator is currently able to handle coupled inequality constraints, to the best of our knowledge. For the novel iterative algorithm, conditions for its asymptotic convergence to the optimal – inequality-constrained – state estimates of CMHE are derived, and asymptotic stability of the estimation error is established. As a positive side effect, less conservative convergence conditions are obtained for the unconstrained S-PMHE algorithm reported in Schneider et al. (2013). Finally, the improved performance of the inequality-constrained S-PMHE algorithm is demonstrated on a numerical example.

The remaining part of this paper is organized as follows. Section 2 recalls constrained CMHE and introduces the system partitioning. In Section 3, the iterative, partition-based S-PMHE algorithm is derived. Section 4 presents the main convergence and stability results. The method is illustrated on a numerical example in Section 5 before the paper is concluded in Section 6.

☆ The material in this paper was partially presented at the 12th European Control Conference, July 17–19, 2013, Zurich, Switzerland. This paper was recommended for publication in revised form by Associate Editor Andrey V. Savkin under the direction of Editor Ian R. Petersen.

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http://dx.doi.org/10.1016/j.automatica.2015.08.016
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2. Preliminaries

We study a large-scale physical system of the form

$$x^\diamond(k+1) = Ax^\diamond(k) + w^\diamond(k), \qquad x^\diamond(0) = x_0^\diamond, \tag{1a}$$

$$y^\diamond(k) = Cx^\diamond(k) + v^\diamond(k). \tag{1b}$$

$x^\diamond(k) \in \mathbb{R}^n$ and $y^\diamond(k) \in \mathbb{R}^p$ are the state and output vectors at time index k, while $w^\diamond(k) \in \mathbb{R}^n$ and $v^\diamond(k) \in \mathbb{R}^p$ represent process and measurement disturbances, respectively. Finally, $x_0^\diamond$ refers to the initial condition and $A \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{p\times n}$ are the system and output matrices. The solution of (1) at time k for given $x_0^\diamond$ is denoted as $x^\diamond(k, x_0^\diamond, w^\diamond)$.

In this paper, we make the following assumption:

Assumption 1 (*Observability*)**.** The pair (A, C) is observable.

In Rao et al. (2001), it is proposed to estimate $x^\diamond(k', x_0^\diamond, w^\diamond)$ by solving, in each time step k', the optimization problem

$$\min_{\Delta x(k^0),x,w,v} \quad \Phi \tag{2a}$$

$$\text{s.t. } \Phi = \frac{1}{2}\left\|\Delta x(k^0)\right\|^2_{\tilde{P}(k^0)} + \frac{1}{2}\left(\sum_{k=k^0}^{k'-1} \|w(k)\|^2_{\tilde{Q}} + \|v(k)\|^2_{\tilde{R}}\right) \tag{2b}$$

$$x(k^0) = \bar{x}(k^0) + \Delta x(k^0), \tag{2c}$$

$$x(k+1) = Ax(k) + w(k), \tag{2d}$$

$$y^\diamond(k) = Cx(k) + v(k), \tag{2e}$$

where $k \in \{k^0, \ldots, k'-1\}$. In (2), up to $K = k' - k^0$ measurement samples $y^\diamond(k)$ are processed to obtain the estimates $x = \langle x(k^0), \ldots, x(k')\rangle$, $w = \langle w(k^0), \ldots, w(k'-1)\rangle$ and $v = \langle v(k^0), \ldots, v(k'-1)\rangle$, where the bracket notation $\langle a_1, \ldots, a_m\rangle$ indicates $[a_1^T \ldots a_m^T]^T$ throughout this text. Given an *a priori* estimate of the state at the beginning of the horizon $\bar{x}(k^0)$ and symmetric positive definite weighting matrices $\tilde{P}(k^0) \in \mathbb{R}^{n\times n}$, $\tilde{Q} \in \mathbb{R}^{n\times n}$ and $\tilde{R} \in \mathbb{R}^{p\times p}$, the solution $x(k')$ is the desired *a priori* estimate of $x^\diamond(k', x_0^\diamond, w^\diamond)$.

Problem (2a)–(2e) is known as the *unconstrained centralized moving horizon estimation* problem (CMHE). Sometimes, however, additional information is available about system (1). For example, states representing a concentration of some substance in a chemical reactor are always non-negative. Such information can naturally be incorporated into (2) through additional constraints of the form

$$x(k) \in \mathbb{X}, \qquad w(k) \in \mathbb{W}, \qquad v(k) \in \mathbb{V}, \quad \forall k, \tag{2f}$$

where $\mathbb{X}$, $\mathbb{W}$ and $\mathbb{V}$ are polyhedral and convex sets described by the inequality constraints

$$\mathbb{X} = \{x(k) : D^x x(k) + d^x \le 0\},$$
$$\mathbb{W} = \{w(k) : D^w w(k) + d^w \le 0\},$$
$$\mathbb{V} = \{v(k) : D^v v(k) + d^v \le 0\},$$

and $0 \in \mathbb{W}$, $0 \in \mathbb{V}$. The consideration of such inequality constraints is a particular strength of MHE and may lead to improved estimation performance. However, a poor choice of the constraint sets may prohibit stability of the estimation error. This issue is complex and has been discussed in great detail in Rao (2000) and Rao, Rawlings, and Mayne (2003). To avoid such complications, we make the following assumption (Rao et al., 2003).

Assumption 2 (*Feasibility*)**.** The process and measurement disturbances acting on the physical system satisfy $w^\diamond(k) \in \mathbb{W}$ and $v^\diamond(k) \in \mathbb{V}$ for all $k \ge 0$. Furthermore, the disturbances $w^\diamond(k)$ and initial condition $x_0^\diamond$ are such that $x^\diamond(k, x_0^\diamond, w^\diamond) \in \mathbb{X}$ for all $k \ge 0$. In other words, the variables of the physical system are always a feasible solution to problem (2).

Problem (2a)–(2f) is called *constrained CMHE*. Assumption 2 guarantees that a feasible solution exists. Hence, it suffices to check constraint qualifications only for *consistent* active sets, which refer to active sets where the corresponding feasible set is nonempty. Together with Assumption 2, the following assumption will guarantee uniqueness of an optimal solution.

Assumption 3 (*QP Properties*)**.** The weights $\tilde{P}(k^0)$, $\tilde{Q}$ and $\tilde{R}$ are symmetric positive definite for all $k^0 \ge 0$. Furthermore, the constraints (2c)–(2f) satisfy the linear independence constraint qualifications (LICQ) for all consistent active sets.

Lemma 4 (*Uniqueness*)**.** *Under Assumptions 2 and 3, constrained CMHE (2) has a unique minimizer.*

Proof. If we introduce the vector $u = \langle \Delta x(k^0), w, v\rangle$ and define $H, G_{ij}, \gamma_i, i = 1, 2, 3, j = 1, 2$, accordingly, then problem (2a)-(2f) can be rewritten as

$$\min_{u,x} \quad \frac{1}{2} u^T H u \tag{3a}$$

$$\text{s.t.} \quad G_{11} u + G_{12} x = \gamma_1, \tag{3b}$$

$$G_{21} u + G_{22} x = \gamma_2, \tag{3c}$$

$$G_{31} u + G_{32} x \le \gamma_3. \tag{3d}$$

Here, Eqs. (3b), (3c) and (3d) represent the constraints (2c)–(2d), (2e) and (2f), respectively. In Eq. (3a), the matrix $H = \text{diag}(\tilde{P}, \tilde{Q}, \ldots, \tilde{Q}, \tilde{R}, \ldots, \tilde{R})$ inherits its positive-definiteness from the weighting matrices $\tilde{P}$, $\tilde{Q}$ and $\tilde{R}$ (Assumption 3). In particular, problem (3) is a convex quadratic program. According to its construction from Eqs. (2c) and (2d), the matrix $G_{12} \in \mathbb{R}^{(Kn)\times(Kn)}$ in Eq. (3b) is regular, so that $x \in \mathbb{R}^{Kn}$ can formally be eliminated from Eq. (3b). After this elimination we obtain an equivalent *strictly* convex quadratic program that has, because we assume the existence of a feasible point (Assumption 2), a unique solution. □

Often, large-scale systems are composed of interacting subsystems. Examples for such subsystems are the process units in a chemical process system or in a hydro power plant. Especially, if these interacting subsystems are geographically separated, it is worthwhile to consider alternatives to centralized state estimation approaches. One such alternative is so-called partition-based state estimation, where the state of each subsystem is interpreted as a partition of the state vector of the overall large-scale system. In formal terms, we assume that the large-scale system (1) consists of N interacting subsystems $i \in \mathcal{N} = \{1, \ldots, N\}$. Using the bracket notation introduced earlier, all vectors are partitioned analogously to the state vector as $x(k) = \langle x_1(k), \ldots, x_N(k)\rangle$. Similarly, the matrices have the decompositions $A = [A_{ij}]_{i,j\in\mathcal{N}}$, $C = [C_{ij}]_{i,j\in\mathcal{N}}$, $\tilde{P} = [\tilde{P}_{ij}]_{i,j\in\mathcal{N}}$, $\tilde{Q} = [\tilde{Q}_{ij}]_{i,j\in\mathcal{N}}$ and $\tilde{R} = [\tilde{R}_{ij}]_{i,j\in\mathcal{N}}$ where the submatrices have the dimensions $A_{ij} \in \mathbb{R}^{n_i\times n_j}$, $C_{ij} \in \mathbb{R}^{p_i\times n_j}$, $\tilde{P}_{ij} \in \mathbb{R}^{n_i\times n_j}$, $\tilde{Q}_{ij} \in \mathbb{R}^{n_i\times n_j}$ and $\tilde{R}_{ij} \in \mathbb{R}^{p_i\times p_j}$. This partitions the equality constraints (2c)–(2e) depending on which subsystem each state or measurement on the left hand side of these equations physically belongs to. The inequality constraints (2f) may also contain variables of different subsystems, however, their assignment to a particular subsystem is a design choice. Without loss of generality, we assume in the following that the row order of the matrices and vectors D^x, D^w, D^v, d^x, d^w, and d^v already reflects a particular design choice, in the sense that their corresponding (i, j)-blocks completely describe all inequalities that have been assigned to subsystem i. For systems partitioned in this way, a state estimation algorithm is presented next.

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