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Errors-in-variables identification in dynamic networks — Consistency results for an instrumental variable approach☆

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ABSTRACT

In this paper we consider the identification of a linear module that is embedded in a dynamic network using noisy measurements of the internal variables of the network. This is an extension of the errors-in-variables (EIV) identification framework to the case of dynamic networks. The consequence of measuring the variables with sensor noise is that some prediction error identification methods no longer result in consistent estimates. The method developed in this paper is based on a combination of the instrumental variable philosophy and closed-loop prediction error identification methods, and leads to consistent estimates of modules in a dynamic network. We consider a flexible choice of which internal variables need to be measured in order to identify the module of interest. This allows for a flexible sensor placement scheme. We also present a method that can be used to validate the identified model.

1. Introduction

Many systems in engineering can be modeled as dynamic networks, as e.g. power systems, telecommunication systems, and distributed control systems. Models of these networks are important either for prediction, simulation, controller design or fault detection. Since sensors are becoming more ubiquitous and cheaper the result is that data can be collected from many variables in a dynamic network, and a system identification approach for modeling particular modules in the dynamic network, becomes attractive. Using this approach it is important to be mindful of the fact that every measurement is contaminated with *sensor noise*.

The literature on dynamic network identification can be split into two categories based on whether the interconnection structure of the network is known or not. In the case that the interconnection structure is not known, the network structure together with the dynamics typically needs to be estimated. The majority of the papers in this category are based on the concept of *Granger Causality* (Granger, 1980). In Caines and Chan (1975) and Gevers and Anderson (1981) it is shown that it is possible to distinguish between open and closed-loop data generating systems. The reasoning is extended to more complex interconnection structures using a non-parametric approach (Materassi & Innocenti, 2010; Materassi & Salapaka, 2012); using a Bayesian approach (Chuiso & Pillonetto, 2012); and using a parametric approach supplemented by ℓ_0 regularization (Seneviratne & Solo, 2012; Yuan, Stan, Warnick, & Goncalves, 2011), ℓ_1 regularization (Friedman, Hastie, & Tibshirani, 2010), and compressed sensing (Sanandaji, Vincent, & Wakin, 2012). In these papers it is assumed that each node in the network is driven by an unknown, independent stochastic process, each variable is measured without sensor noise, and every variable in the network is measured. It is shown that under these conditions topology detection is possible.

For many networks in engineering, the interconnection structure is known. This knowledge can be incorporated in the identification problem. A type of interconnection structure that results from the discretization of partial differential equations is a *spatially distributed system* where each node is connected only to its direct neighbors. Methods for identifying such systems are presented in Ali, Popov, Werner, and Abbas (2011), Haber and Verhaegen (2012) and Massioni and Verhaegen (2008), where common assumptions

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are that each subsystem is identical, known external excitation signals are present at each node, and no process noise is present in the networks. Because such networks are typically very large, emphasis is on improving computational speed of the identification algorithms.

Identification in networks with a general interconnection structure have been investigated in Dankers, Van den Hof, Heuberger, and Bombois (in press) and Van den Hof, Dankers, Heuberger, and Bombois (2013), where methods are presented to consistently identify a single transfer function embedded in a dynamic network. It is shown that by knowing the interconnection structure assumptions on the correlation of process noise can be relaxed, and that there is considerable flexibility in which variables need to be measured. In these papers the measurements are assumed to be sensor noise free.

Variance issues of identified models in a structured network have been addressed in Everitt, Hjalmarsson, and Rojas (2013, 2014), Gunes, Dankers, and Van den Hof (2014) and Wahlberg, Hjalmarsson, and Mårtensson (2009) where it is shown that "extra" measurements can be used to reduce the variance of the estimated transfer function. These papers assume that there is no process noise, and known external excitation and sensor noise are both present.

In this paper we consider a very general framework that covers all the cases discussed in the previous literature review, where there may or may not be known external excitation present, there is both (correlated) process noise and (correlated) sensor noise present, the modules making up the network are not identical, and not all internal variables of the network are measurable. Moreover, we do not make assumptions on the whiteness of the sensor noise. The main assumption that we make is that the interconnection structure of the network is known. We address the following question: under what conditions is it possible to consistently identify a particular module embedded in a dynamic network when only noisy measurements of a subset of the internal variables of the network are available? This is an extension of the so-called Errors-in-Variables (EIV) framework to the case of dynamic networks.

In the system identification literature, the open loop EIV problem has been extensively studied, see e.g. Söderström (2007, 2012). The main conclusion in these papers is that either prior knowledge about the system or a controlled experimental setup is required to ensure consistent estimates. This latter condition concerns either periodic excitation (Pintelon & Schoukens, 2012; Schoukens, Pintelon, Vandersteen, & Guillaume, 1997; Söderström & Hong, 2005) or repeated experiments Schoukens et al. (1997) and Pintelon and Schoukens (2012). The closed-loop EIV problem has been studied in Pintelon and Schoukens (2012) and Söderström, Wang, Pintelon, and Schoukens (2013) where it is shown that the plant is identifiable if a noise-free and sufficiently exciting reference signal is available.

In the extension of this problem to the dynamic network case fruitful use can be made of additionally measured signals in the network that can serve as instrumental variables, thereby enabling a considerable simplification of the EIV problem. The method presented in this paper is based on *instrumental variable* (IV) reasoning. The IV method was developed in the econometrics field (Wright, 1928), where the method has been applied to static networks (*structural equation models* in statistics) (Angrist, Imbens, & Rubin, 1996). In the econometrics literature IV methods are recognized to have three main advantages when aiming to obtain consistent estimates:

(**1**). Presence of sensor noise on the input (*explanatory variable* in economics) is no problem (Durbin, 1954);

(**2**). Confounding variables (*omitted variables* in econometrics), i.e. unknown or unmeasured variables for which there is a path to both the output and an input, are no problem (Angrist & Krueger, 2001; Becker, 2010);

(**3**). Presence of algebraic loops in the data generating system (*simultaneity* in econometrics) is no problem (Becker, 2010).

In this paper we show that the same advantages can be converted to the situation of a dynamic network, and moreover that the choice of candidate instrumental variable signals actually can be widened.

In the system identification literature IV methods are also extensively used for identification in open-loop (Söderström & Stoica, 1983; Wong & Polak, 1967), and closed-loop systems (Gilson & Van den Hof, 2005; Söderström & Stoica, 1989; Söderström, Stoica, & Trulsson, 1988). Again, IV methods have been recognized to be robust to the presence of (particular) sensor noise on the input (Söderström & Hong, 2005; Thil & Gilson, 2011).

In this paper we generalize the IV method such that it is possible to obtain consistent estimates of a transfer function embedded in a dynamic network where all predictor inputs are measured with (colored) sensor noise, and the instrumental signal(s) are contaminated too.

In Section 2 background material on dynamic networks, prediction error identification and IV methods is presented. In Sections 3 and 4 the main result is presented for two different cases of IV signals. In Section 5 results are generalized for a flexible choice of predictor inputs, while in Section 6 a practical implementation of the method is proposed. In Section 7 a method is presented to validate the obtained model.[1]

2. Background

2.1. Dynamic networks

The specific identification framework considered in this paper is based on Van den Hof et al. (2013). A dynamic network is built up of L elements, related to L scalar *internal variables* w_j, $j = 1, \dots, L$. Each internal variable is defined by:

$$w_j(t) = \sum_{k \in \mathcal{N}_j} G^0_{jk}(q) w_k(t) + r_j(t) + v_j(t) \quad (1)$$

where G^0_{jk}, $k \in \mathcal{N}_j$ is a proper rational transfer function, q^{-1} is the delay operator, i.e. $q^{-1}w_j(t) = w_j(t-1)$ and,

- $\mathcal{N}_j$ is the set of indices of internal variables that are direct inputs to the transfer functions determining w_j, i.e. $k \in \mathcal{N}_j$ iff $G^0_{jk} \neq 0$;
- v_j is *process noise*, that is modeled as a realization of a stationary stochastic process with rational spectral density: $v_j = H^0_j(q)e_j$ where e_j is a white noise process, and H^0_j is a monic, stable, minimum phase transfer function;
- r_j is an *external variable* that is known to the user, and may be manipulated by the user.

It may be that the noise and/or external variables are not present at some nodes. The network is defined by:

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & G^0_{L-1\,L} \\ G^0_{L1} & \cdots & G^0_{L\,L-1} & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_L \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_L \end{bmatrix},$$

where G^0_{jk} is non-zero if and only if $k \in \mathcal{N}_j$ for row j, and v_k (or r_k) is zero if it is not present. Using an obvious notation this results in

[1] This paper is based, in part, on the preliminary results of Dankers, Van den Hof, Bombois, and Heuberger (2014).