Automatica 62 (2015) 184–192

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Local optimization of dynamic programs with guaranteed satisfaction of path constraints[☆]

Jun Fu[a], Johannes M.M. Faust[c], Benoît Chachuat[b], Alexander Mitsos[c,1]

[a] Department of Mechanical Engineering, Massachusetts Institute of Technology (MIT), Cambridge, MA, 02139, USA

[b] Centre for Process Systems Engineering, Department of Chemical Engineering, Imperial College, London, SW7 2AZ, UK

[c] AVT Process Systems Engineering (SVT), RWTH Aachen University, Turmstrasse 46, 52064 Aachen, Germany

ARTICLE INFO

Article history:
Received 15 August 2014
Received in revised form
13 June 2015
Accepted 30 August 2015
Available online 11 November 2015

Keywords:
Dynamic optimization
Path constraints
Semi-infinite programs
Optimization with dynamics embedded
Optimal control

ABSTRACT

An algorithm is proposed for locating a feasible point satisfying the KKT conditions to a specified tolerance of feasible inequality-path-constrained dynamic programs (PCDP) within a finite number of iterations. The algorithm is based on iteratively approximating the PCDP by restricting the right-hand side of the path constraints and enforcing the path constraints at finitely many time points. The main contribution of this article is an adaptation of the semi-infinite program (SIP) algorithm proposed in Mitsos (2011) to PCDP. It is proved that the algorithm terminates finitely with a guaranteed feasible point which satisfies the first-order KKT conditions of the PCDP to a specified tolerance. The main assumptions are: (i) availability of a nonlinear program (NLP) local solver that generates a KKT point of the constructed approximation to PCDP at each iteration if this problem is indeed feasible; (ii) existence of a Slater point of the PCDP that also satisfies the first-order KKT conditions of the PCDP to a specified tolerance; (iii) all KKT multipliers are nonnegative and uniformly bounded with respect to all iterations. The performance of the algorithm is analyzed through two numerical case studies.

1. Introduction

Dynamic optimization refers to mathematical programs whereby the objective and constraint functions depend on the solution of differential or difference equations. Dynamic optimization has been widely applied in chemical engineering (Biegler, 2010; Srinivasan, Palanki, & Bonvin, 2003), mechanical engineering (Hussein & Bloch, 2008; Shin & McKay, 1986), aerospace engineering (Bainum & Kumar, 1980) and other disciplines (Floudas et al., 1999). Constrained dynamic optimization problems are practically important, e.g., to enforce product quality or to guarantee safety (Feehery & Barton, 1998; Srinivasan et al., 2003). Constraints fall in either one of two categories, namely point constraints and path constraints. The former are usually expressed as functions of the states at the end of time horizon, whereas the latter are functions of the states and/or controls over the entire time horizon. The focus of this article is on dynamic optimization with path constraints. Point constraints, which do not pose any further complication for the approach herein, are omitted for simplicity. Throughout the article, it is assumed that a control vector parameterization has been performed, i.e., a finite number of decision variables is assumed.

Numerical solution methods for such dynamic optimization problems rely on nonlinear programming (NLP) techniques, either with or without parameterization of the state trajectories. In the simultaneous method, also known as orthogonal collocation approach (Betts & Huffman, 1992; Biegler, 2007; Tsang, Himmelblau, & Edgar, 1975), the state trajectories are parameterized and the residuals of the differential equations are enforced as constraints at specified collocation times. In the sequential method (Biegler, 2010; Goh & Teo, 1988), the state trajectories are regarded as functions of the control decision variables. In the direct multiple shooting method (Bock & Plitt, 1984), the state trajectories are formed by piecing together those of finite single shooting problems on the corresponding subintervals over which the parameterized control is applied (see p. 243 in Bock & Plitt, 1984).

☆ Dr. Jun Fu's work was partially supported by Natural Sciences and Engineering Research Council of Canada (NSERC) grant 387509. Prof. Mitsos would like to acknowledge funding from the DFG-Cluster grant DFG-PAK 635/1 Project MA 1188/34-1 "Strategie zur robusten dynamischen Echtzeitoptimierung" and discussions within that cluster. The material in this article was not presented at any conference. This paper was recommended for publication in revised form by Editor Berç Rüstem.

E-mail addresses: junfu@mit.edu, fujuncontrol@gmail.com (J. Fu), johannes.faust@avt.rwth-aachen.de (J.M.M. Faust), b.chachuat@imperial.ac.uk (B. Chachuat), amitsos@alum.mit.edu (A. Mitsos).

1 Tel.: +49 2418094704; fax: +49 2418092326.

http://dx.doi.org/10.1016/j.automatica.2015.09.013
0005-1098/

Techniques for dealing with inequality path constraints have been developed for the three methods, e.g., Biegler (2010), Bock and Plitt (1984), Dai and Cochran (2009), Feehery and Barton (1998), Fikar (2001), Goh and Teo (1988), Jacobson and Lele (1969), Li, Yu, Teo, and Duan (2011), Loxton, Teo, Rehbock, and Yiu (2009), Parida and Raha (2009), Schlegel, Stockmann, Binder, and Marquardt (2005), Teo, Rehbock, and Jennings (1993), Vassiliadis, Sargent, and Pantelides (1994) and White, Perkins, and Espie (1996). The common feature of these techniques is that the path constraints are (explicitly or implicitly) enforced at finitely many points only. Particularly popular are discretization of the path constraints as interior-point constraints and transcription as integral constraint, possibly used in combination (Vassiliadis et al., 1994). The former method enforces the path constraint at a finite number of time points, so constraint violation can occur at any point other than those where the constraints are enforced. The latter one enforces a time-integral of the constraint violation as a constraint, which is allowed to be less than or equal to a small positive constant for regularity reasons, therefore allowing for small violations along the time horizon, too. Particularly relevant to this article are the works by Chen and Vassiliadis (2005) and by Potschka, Bock, and Schlöder (2009). Chen and Vassiliadis (2005) presents an algorithm solving path-constrained optimal control problems, yet violation of the path constraints by a small amount cannot be prevented for a finite number of iterations. Potschka et al. (2009) develops an algorithm solving path-constrained optimal control problems (without proof of convergence), but to the authors' best knowledge does not achieve both guaranteed rigorous satisfaction of path constraints and finite convergence. More recently, Zhao and Stadtherr (2011) have described an algorithm capable of locating an ϵ-estimated global optimum of path-constrained dynamic systems with guaranteed satisfaction of the path constraints, but this rigorous algorithm uses a deterministic global optimization approach, and as such it is currently applicable to problems with a small number of degrees of freedom only. Note that indirect methods can be used for the continuous optimal control problem under the assumption that the switching structure of the path constraint is known (Hannemann-Tamás & Marquardt, 2012). Note also that the α method in Peter, Parida, and Raha (2010) can be used for infinite dimensional problems subject to the regularization assumptions. With the exception of Zhao and Stadtherr (2011), to our best knowledge, *none of the existing methods can guarantee rigorous satisfaction of path constraints over the entire time horizon within a finite number of iterations*. It is the focus of this article to develop an algorithm for path-constrained dynamic optimization problems that relies on local optimization techniques, while coming with a certification of feasibility for the path constraints.

An important class of optimization problems are semi-infinite programs (SIP), namely optimization problems with a finite number of decision variables but an infinite number of constraints. For theoretical developments and applications of SIP, we refer the reader to reviews (Hettich & Kortanek, 1993; Polak, 1987) and latest results (Mitsos, 2011; Mitsos & Tsoukalas, 2015; Stein & Steuermann, 2012). In the context of path-constrained dynamic optimization, SIP formulations arise naturally if time is viewed as the (single) parameter of SIP (Loxton et al., 2009; Sachs, 1998). Through this connection, the work by Chen and Vassiliadis (2005) can be seen as an adaptation of the SIP algorithm of Blankenship and Falk (1976) to path-constrained dynamic optimization. The work by Potschka et al. (2009) is essentially a first combination of local reduction method of SIP (Hettich & Kortanek, 1993) with the idea of Blankenship and Falk (1976) in the framework of the direct multiple shooting method.

This article develops an algorithm for locating a feasible point satisfying the KKT conditions to a specified tolerance of semi-infinite-dimensional, inequality-path-constrained dynamic programs (PCDP). Based on the right-hand restriction method proposed in Mitsos (2011) for standard SIP, the algorithm proceeds by iteratively approximating the PCDP by restricting the right-hand side of the path constraint and enforcing it at a finite number of time points. A dynamic optimization problem with *finitely* many constraints is solved to local optimality at each iteration, thereby making it possible to combine it with state-of-the-art local dynamic optimization codes. It will be established that the algorithm terminates finitely with a guaranteed feasible point and a certificate of satisfaction of the first-order KKT conditions of the PCDP to a specified tolerance under the following main assumptions: (i) availability of a nonlinear program (NLP) local solver that generates a KKT point of the constructed approximate PCDP at each iteration if this problem is indeed feasible; (ii) existence of a Slater point of the PCDP that also satisfies the first-order KKT conditions of the PCDP to a specified tolerance; and (iii) KKT multipliers are nonnegative and uniformly bounded with respect to all iterations.

The remaining part of the article is organized as follows. Section 2 states the path-constrained dynamic optimization problems of interest, where for simplicity a single constraint is considered. Section 3 describes the algorithm to locate a feasible approximate KKT point of the path-constrained dynamic optimization problem with guaranteed satisfaction of path constraints, and it also presents a proof of finite convergence of the algorithm. Section 4 illustrates the property of guaranteed satisfaction of path constraints and analyzes the effect of tuning parameters in the algorithm using two numerical case studies. Section 5 presents conclusions and an outlook on future work.

2. Problem statement

We consider semi-infinite-dimensional, inequality-path-constrained dynamic optimization problems of the form:

$$\begin{aligned} \min_{u \in U} \quad & S(x(t_f, u)) \\ \text{s.t.} \quad & g(x(t,u),u) \le 0, \quad \forall t \in T, \\ & \dot{x}(t,u) = f(x(t,u),u), \quad \forall t \in T, \\ & x(t_0,u) = x_0(u), \end{aligned} \tag{PCDP}$$

where $t \in T := [t_0, t_f]$ represents the independent variable, e.g., time; $u \in U$ denote the time-invariant control/decision variables, with $U \subset \mathbb{R}^n$ nonempty and compact; and $x(\cdot, u)$ is the state response to a given control u, with $x(t,u) \in X, \forall (t,u) \in T \times U$ and $X \subset \mathbb{R}^{n_x}$ nonempty and compact. The objective function $S : X \to \mathbb{R}$, path-constraint function $g : X \times U \to \mathbb{R}$, right-hand-side function $f : X \times U \to \mathbb{R}^{n_x}$, and initial-value function $x_0 : U \to \mathbb{R}^{n_x}$ are all assumed to be continuously differentiable in their respective arguments. No convexity assumptions are made, but local solutions are considered.

Remark 1. Optimal control problems with control trajectories as their decision variables can be approximated (restricted) into (PCDP) via the control vector parameterization technique (Biegler, 2010; Lin, Loxton, & Teo, 2014; Loxton, Lin, Rehbock, & Teo, 2012; Teo, Goh, & Wong, 1991). Moreover, problems with an integral term as part of their objective function or with explicit time dependence can be transformed into (PCDP) via the introduction of extra variables and equations in the dynamic system (Chachuat, 2006–2007; Teo et al., 1991).

The main objective of this article is to develop an algorithm to obtain a feasible point satisfying the KKT conditions of (PCDP) to a specified tolerance.