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Quasi-synchronization of heterogeneous dynamic networks via distributed impulsive control: Error estimation, optimization and design[☆]

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This paper studies leader–follower synchronization in heterogeneous dynamic networks via distributed impulsive control. The goal is to drive the followers to approximately synchronize with the leader within a nonzero error bound, referred to as quasi-synchronization. Some fundamental and yet challenging problems are addressed : (1) How to obtain a tight quasi-synchronization error bound; (2) How many and which nodes should be controlled; (3) How to design the coupling strength, to select the pinned nodes and to determine the impulse intervals to optimize the error bound or to achieve quasi-synchronization within a prescribed error bound. First, some general quasi-synchronization conditions are derived with explicit expressions of the error bound. Second, the pinning strategy and the coupling strength in achieving quasi-synchronization are explored and expressed in term of impulse intervals. It is interesting to find that a large coupling strength will destroy synchrony, in contrast to continuous pinning control. Furthermore, the common requirement that the leader has a directed path to every node is needed only in the case that the impulse intervals can be arbitrarily small. A new concept, referred to as impulse pinning controllability, is introduced to the case that impulses cannot take place too frequently. Third, an optimization of the error bound is discussed and formulated. Under a prescribed error bound, the design problem of the coupling strength, pinned nodes and impulse intervals is solved. Finally, three examples are given to verify the theoretical results.

1. Introduction

Recent years have witnessed an increasing interest in cooperative behaviors of interconnected dynamic systems from science and technology communities. As a typical dynamic process, synchronization has received considerable attention due to its wide occurrence in biological systems, neuronal networks and physiological processes (Pikovsky, Rosenblum, & Kurths, 2001) and its engineering applications, such as consensus of mobile autonomous agents (Cao, Yu, Ren, & Chen, 2013), time synchronization of distributed sensor networks (He, Cheng, Shi, & Chen, 2013; He, Cheng, Shi, Chen, & Sun, 2014), formation control of unmanned vehicles and so on.

Much effort has been made to develop effective control strategies to regulate dynamic networks into a synchronous motion, such as continuous feedback control (Guo, Zhao, & Yang, 2014; Hu, Lam, & Liang, 2013), impulsive control (Guan, Liu, Feng, & Jian,

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2012; He, Chen, Han, & Qian, 2015), adaptive control (Li, Ren, Liu, & Fu, 2013; Su, Chen, Wang, & Lin, 2011), and sampled-data control (Guo, Ding, & Han, 2014). Among them, impulsive control method is regarded as an effective control strategy which allows systems to possess discontinuous inputs. Based on the stability theory of impulsive systems, this method provides a promising prospect for solving problems in which systems cannot endure continuous control inputs, or in some application, it is not possible to provide continuous control inputs. Examples include saving rate regulation of a bank and population control of bacteria by instantaneously changing the density of the bactericide (Yang, 2001). In addition, for plants which are impulsive in nature such as mechanical systems with impacts and nanodevices with electron tunneling effects, impulsive control may be the best choice (Yang, 2001). In Guan, Liu, Feng, and Wang (2010), a distributed impulsive controller was proposed for achieving synchronization of a leaderless dynamic network with time-varying delays. In Liu, Liu, Chen, and Wang (2005), robust synchronization in leader–follower networks was accomplished by applying impulsive control to all nodes. However, it is obviously very expensive, and usually not necessary, to impose controllers to all nodes, especially on a large-sized network. One effective approach is to develop an impulsive control technique based on pinning control, which means that only a small number of followers are guided by the leader. Lu, Kurths, Cao, Mahdavi, and Huang (2012) proposed an impulsive pinning strategy for synchronization of stochastic networks, where the proportion of controlled nodes was determined. Later, a single impulsive controller was proved to be effective in stabilizing networks of coupled linear scalar systems (Liu, Lu, & Chen, 2013) and delayed dynamic networks (Zhou, Wu, & Xiang, 2011). An improved technique was developed by Yang, Cao, and Yang (2013) to deal with impulsive pinning synchronization of coupled delayed reaction–diffusion neural networks. Although much progress has been made, the relationship between impulse intervals and the pinning strategy is largely unknown. Moreover, a common feature of the above-mentioned works is that the controller contains only error signals between the pinned nodes and the leader, while neighbor's information is continuously transmitted between consecutive impulses throughout the entire process. Such a protocol is clearly inefficient regarding the network and computation resources.

In practice, interactions among individual systems may take place only at some discrete time instants rather than continuously due to technological constraints of digital implementation or for saving limited energy of communication (Guo, 2010; Guo, Lu, & Han, 2012). Distributed impulsive control, therefore, provides an effective way to do so, which has been used for consensus of second-order multi-agent systems (Guan et al., 2012; Zhang & Zhou, 2013). However, these works restrict their scope within leaderless networks. Thus, designing an efficient pinning control strategy based on distributed impulsive control and further exploring the relationship between impulse sequences and pinning strategy for synchronization of nonlinear dynamic networks stand out as one motivation of the current study.

The design of an impulsive pinning control strategy involves synthesizing a pinning control protocol and an impulse sequence, so as to synchronize a nonlinear dynamic network with a leader node. In contrast to the majority of recent publications on synchronization and consensus of identical systems, this paper deals with heterogeneous dynamic networks, which widely exist in real-life situations. For example, in multiple robot manipulators with Lagrangian dynamics (Bouteraa & Ghommam, 2009), each manipulator has a different inertia matrix with nonlinear terms describing coriolis and centrifugal forces, as well as gravitational forces due to external disturbances. As the heterogeneity may result in possible loss of synchronization and deteriorate the coordinated performance, it is essential to explore synchronous motions in heterogeneous dynamic networks. However, this is much more challenging as such systems cannot be forced to synchronize by static linear controllers. To tackle the difficult issue, the traditional synchronization problem has to be posed in a more general manner (Lunze, 2012). One resolution to such a challenge is output synchronization, as presented in Ding (2013), Kim, Shim, and Seo (2011), Lunze (2012) and Wieland, Sepulchre, and Allgöwer (2011). Another possible way to effectively use the state feedback control is to add extra constraints onto individual agent dynamics or controllers. In Liu and Hill (2011) and Song, Cao, and Liu (2010), an additional control law was applied to compensate the node differences in the synchronizing process. By assuming that all the nonidentical nodes have a common equilibrium (Xiang & Chen, 2007; Zhao, Hill, & Liu, 2011a), or individual agent dynamics on the common time-varying reference state tend to become the same (Zhao, Hill, & Liu, 2011b), complete synchronization can be achieved in a network of nonidentical nonlinear systems. Recently, a discontinuous state feedback controller was applied to drive coupled heterogeneous multi-agent systems into a synchronous motion (Meng, Lin, & Ren, 2013). Quasi-synchronization provides another way to deal with the synchronization problem for heterogeneous dynamic networks using linear state feedback control. The main advantage, compared with Liu and Hill (2011), Song et al. (2010), Xiang and Chen (2007), Zhao et al. (2011a) and Zhao et al. (2011b), is that it removes extra constraints on individual agent dynamics or controllers. Correspondingly, a nonzero bound of the synchronization error needs to be estimated. Hill and Zhao (2008) investigated quasi-synchronization conditions for delay-free systems, where an expression of the error bound was given. An extension to time-delay systems was reported in Zhong, Liu, and Thomas (2012). An adaptive mechanism was constructed in Das and Lewis (2010), where a loose bound was presented. In these results, attentions were paid on quasi-synchronization conditions, while whether the error bound was tight or which factors contributed to the error bound were not addressed. Unlike (Das & Lewis, 2010; Hill & Zhao, 2008; Zhong et al., 2012), He, Du, Qian, and Cao (2013) presented an effective method of deriving the error bound in a leaderless network. A similar idea can also be found in He, Qian, Han, and Cao (2011), He, Qian, Han, and Cao (2012) and Huang, Li, and Liao (2007), where only two chaotic systems were involved. In He, Han, and Qian (2014), synchronization in a leader–follower heterogenous dynamic network under impulsive control was studied. However, the derived matrix inequalities were not easy to be solved and the effect of pinning strategy was not well explored. In addition, how to optimize the error bound and design the impulse sequences were not mentioned. Therefore, a comprehensive analysis of impulsive quasi-synchronization in leader–follower heterogenous dynamic networks is in great demand, which provides another motivation of the current study.

In this paper, we investigate the problem of quasi-synchronization in leader–follower heterogenous dynamic networks by applying distributed impulsive control. The following fundamental issues are addressed: (1) How to find a tight error bound; (2) How many and which nodes should be controlled in order to achieve quasi-synchronization; (3) How to design the coupling strength, pinned nodes and impulse intervals to optimize the error bound or to achieve quasi-synchronization within a prescribed error bound. To proceed, first, a network of nonlinear systems with non-identical parameters is formulated. A time-varying Lyapunov function, inspired by Chen, Wei, and Zheng (2013) and Liu et al. (2005), is used to derive some less-conservative criteria, in which an explicit expression of the error bound is given. Second, by taking some special forms of a general Lyapunov function, several sufficient conditions ensuring quasi-synchronization are obtained in a general network. Then, the effects of the coupling strength and pinning control strategy are further analyzed in terms of the impulse intervals in a symmetric network. Criteria for choosing an