Available online at www.sciencedirect.com

ScienceDirect

Journal of the Franklin Institute 355 (2018) 3122–3144

www.elsevier.com/locate/jfranklin

Global exponential stability for multi-group neutral delayed systems based on Razumikhin method and graph theory

Ying Guo [a,*], Yida Wang [b], Xiaohua Ding [b]

[a] *Department of Mathematics, Qingdao Technological University, Qingdao 266520, PR China*

[b] *Department of Mathematics, Harbin Institute of Technology (Weihai), Weihai 264209, PR China*

Received 22 July 2017; received in revised form 9 December 2017; accepted 16 February 2018
Available online 2 March 2018

Abstract

This paper is concerned with the global exponential stability for an original class called coupled systems of multi-group neutral delayed differential equations (MNDDEs). By employing Razumikhin method along with graph theory, sufficient conditions are established to guarantee the global exponential stability of MNDDEs, which are in the form of Razumikhin theorem. For the convenience of use, sufficient conditions in the form of coefficients are also obtained. Furthermore, coefficient-type criterion is employed to study the stability of coupled neutral delay oscillators which shows the applicability of our findings. Finally, two numerical examples are given to demonstrate the validity and feasibility of the theoretical results.

1. Introduction

It is well known that, in the real world, many phenomena can be modeled by neutral differential equations (NDEs) which play an important role not only in control models [1], but also in population models [2]. Thus, NDEs are widely employed to many areas such as economics, biology, physics, mechanics. Furthermore, many attractive results have been

* Corresponding author.
E-mail address: mathgy@126.com (Y. Guo).

https://doi.org/10.1016/j.jfranklin.2018.02.010

obtained (See [3–8] and the references therein). The properties of NDEs have drawn considerable attention of researchers, among which emphasis is placed on the stability that served as a main character of a system. It is taken for granted that the stability analysis for NDEs has earned the interest of an increasing number of scientists [9–12].

As is well known to all that time delays often exist in many practical situations. For example, in control systems, signals transmitting from the sensor to the controller and then from the controller to the actuator, can induce time delays. Again for instance, in epidemiology, there usually exists an interval between a man being infected and getting ill. It is worth noting that time delays can affect the dynamic behaviors of a system, such as instability, oscillation, divergence, and chaos [13–15]. Therefore, considering the effect of time delays to the dynamics of NDEs, neutral delayed differential equations (NDDEs) have been widely studied [16,17].

On the other hand, during the past few years, multi-group models have been an active research topic, because it holds wide applications in lots of fields comprising mathematical ecology and especially in epidemiology [18,19]. For example, multi-group models have been used to describe the dynamic transmission of many infectious diseases in heterogeneous host populations, such as measles, mumps, and HIV/AIDS [20–23]. Multi-group models have started to attract people's interest since it is more suitable for realistic situations. A lot of investigations about multi-group models have been reported, however, in the scope of authors' knowledge, there are few researches for multi-group neutral delayed differential equations (MNDDEs).

Except Lyapunov method, as an efficient tool to study the stability, Razumikhin method is also commonly used and has an outstanding advantage from the perspective of authors, i.e., we are able to utilize simple functions rather than functionals, which implies the imposed conditions of Razumikhin method have fewer limitations. During the past few years, Razumikhin-type stability theorem for various kinds of single-group models was established, such as delayed models [24,25], discrete models [26], and stochastic models [27,28]. According to the references mentioned above, it is easy to know that Razumikhin method brings a powerful tool to study the stability of delayed differential equations. To the best of authors' knowledge, Razumikhin method has never been applied to MNDDEs so far. This paper fills this gap.

In recent years, coupled systems have been extensively studied and also have widespread applications developed in mechanical, electronic, and biological fields [29–32]. All of these applications mainly depend on the dynamical characteristics. Studying the dynamic behaviors of coupled systems, especially the stability, is one of the dominant themes. The Lyapunov method is a feasible method as well as a significant tool in the theory of stability. However, for special coupled systems, it is quite tough to consider the direct construction of an appropriate Lyapunov function, because their dynamics base on both the individual vertex-dynamics and the coupling topology. Fortunately, a novel approach combined with graph theory was proposed to construct a Lyapunov function, which was firstly applied to the investigation for stability of coupled systems by Li et al.[33,34]. By using this method, the global stability for some classes of coupled systems have been studied successfully, see [35–40].

Due to the growing ties among people in different countries and regions, absolutely independent places are hard to find, so it is the same with ecological communities. For example, dispersal is an important factor determining community composition and community turnover. In order to give a better description of some situations, dispersal should be incorporated into multi-group models. The majority of researches are based on a single digraph