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Brief paper

Self-triggered continuous–discrete observer with updated sampling period[☆]

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ABSTRACT

This paper deals with the design of high gain observers for a class of continuous-time dynamical systems with discrete-time measurements. Different approaches based on high gain techniques have been followed in the literature to tackle this problem. Contrary to these works, the measurement sampling time is considered to be variable. Moreover, the new idea of the proposed work is that the use of the output measurements by the observer follows an event based on an extended observer state component. Assuming that the vector fields related to the considered system are globally Lipschitz, the asymptotic convergence of the observation error is established. As an application of this approach, a state estimation problem of an academic bioprocess is studied, and its simulation results are discussed.

1. Introduction

Estimating the state of a partially measured dynamical system is a classical problem in control theory. An algorithm that solves this problem is an asymptotically convergent observer. When the measurement is available only at some discrete-time instant, a continuous–discrete time observer has to be designed. The study of this type of algorithm can be traced back to Jazwinski who introduced the continuous–discrete Kalman filter to solve a filtering problem for stochastic continuous–discrete time systems (see Jazwinski, 1970). Inspired by this approach, the continuous–discrete high-gain observer has been studied in Deza, Busvelle, Gauthier, and Rakotopara (1992). Since then, different approaches have been investigated. The robustness of an observer with respect to time discretization was studied in Arcak and Nešić (2004) (see also Postoyan & Nešić, 2012). In Moraal and Grizzle (1995), a Newton observer is provided which estimates the state at time t_k from N consecutive measurements of outputs and inputs; in Bıyık and Arcak (2006), the authors show how this method can be implemented in the case where the sampled system is not known analytically. In Karafyllis and Kravaris (2009) observers were designed from an output predictor (see also related works in Ahmed-Ali, Van Assche, Massieu, and Dorleans (2013)). Some other approaches based on time delayed techniques have also been considered in Raff, Kogel, and Allgower (2008). Recently, a new continuous–discrete observer design methodology for Lipschitz nonlinear systems based on reachability analysis was presented in Dinh, Andrieu, Nadri, and Serres (2015) (see also Mazenc, Andrieu, & Malisoff, 2015).

In this note, we also consider the design of a continuous–discrete time observer. However, in opposition to these results, we consider the case in which the sampling time is variable and used as a tuning parameter. More precisely, we consider that the quantity $t_{k+1} - t_k$ is a part of the design of the continuous–discrete

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observer. Hence, in the proposed algorithm, the measurement time is computed online. In fact, the use of sensors follows an event based on an extended observer state component. This may be related to the event-triggered control methodology (see for instance Seuret & Prieur, 2011; Tabuada, 2007).

In high-gain designs, the asymptotic convergence of the estimate to the state is obtained by dominating the Lipschitz nonlinearities with high-gain techniques. However, there is a trade-off between the high-gain parameter and the measurement step size. This can lead to restrictive design conditions on the sampling measurement time (see also Nadri, Hammouri, & Grajales, 2013). Inspired by Andrieu, Praly, and Astolfi (2009), the extra observer state component estimates the local Lipschitz constant (roughly speaking $\frac{|\dot{x}_a-\dot{x}_b|}{|x_a-x_b|}$) in order to maximize the measurement sampling interval.

The paper is organized as follows. The class of systems considered and the structure of the estimation algorithm are given in Section 2. The main result and its proof are given in Section 3. Section 4 contains an illustrative example. Finally, Section 5 is devoted to the conclusion.

2. Problem statement and structure of the observer

2.1. Class of systems considered

In this work we consider the problem of designing an observer for nonlinear systems that are diffeomorphic to the following form:

$$\dot{x} = Ax + f(x, u), \tag{1}$$

where the state x is in $\mathbb{R}^n$, $u : \mathbb{R} \to \mathbb{R}^p$ is a known input in the space of essentially bounded measurable functions from $\mathbb{R}_+$ to $\mathbb{R}^p$ (denoted $\mathbb{L}^\infty(\mathbb{R}_+, \mathbb{R}^p)$), A is a matrix in $\mathbb{R}^{n\times n}$ and $f : \mathbb{R}^n \times \mathbb{R}^p$ is a locally Lipschitz vector field both having the following triangular structure:

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \\ 0 & \cdots & \cdots & 0 & 1 \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}, \qquad f(x,u) = \begin{bmatrix} f_1(x_1, u) \\ f_2(x_1, x_2, u) \\ \vdots \\ f_n(x, u) \end{bmatrix}.$$

The measured output is given as a sequence of values $(y_k)_{k\geqslant 0}$ in $\mathbb{R}$

$$y_k = Cx(t_k), \tag{2}$$

where $(t_k)_{k\geqslant 0}$ is a sequence of times to be selected and $C = [1\,0\,\cdots\,0]$ is in $\mathbb{R}^n$. In this paper, we shall denote by $\langle\cdot,\cdot\rangle$ the canonical scalar product in $\mathbb{R}^n$ and by $\|\cdot\|$ the induced Euclidean norm; we shall use the same notation for the corresponding induced matrix norm. Also, we use the symbol $'$ to denote the transposition operation.

We consider the case in which the vector field $f : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n$ satisfies the following assumption.

Assumption 1. The function $f = (f_1, \dots, f_n)'$ is such that the following incremental bound is satisfied for all $(x, e, u) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p$,

$$|f_j(x+e, u) - f_j(x, u)| \leqslant c(x, u) \sum_{i=1}^{j} |e_i|, \tag{3}$$

where $c : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}_+$ is a continuous function which satisfies the following bound

$$c(x, u) \leqslant \Gamma(u), \quad \forall\, (x, u) \in \mathbb{R}^n \times \mathbb{R}^p, \tag{4}$$

where $\Gamma : \mathbb{R}^p \to \mathbb{R}_+$.

Compared to the preliminary version of this work presented in Andrieu, Nadri, Serres, and Vivalda (2013), now a larger class of nonlinear systems is addressed. Indeed, general upper triangular systems are now allowed.

Note that in the case in which we know a bound on the input u, we come back to the globally Lipschitz context. However, even in this case, we believe that employing a tighter bound in term of a state-dependent function c implies that the sensors are less used than they would be if we were considering directly the Lipschitz bound.

2.2. Updated sampling time observer

The continuous–discrete time observer with updated sampling period is given by[1]

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + f(\hat{x}(t), u(t)), \quad t \in [t_k, t_{k+1}) \\ \hat{x}(t_{k+1}) = \hat{x}(t_{k+1}^-) + \delta_k \mathcal{L}(t_{k+1}^-) K (C\hat{x}(t_{k+1}^-) - y_{k+1}), \end{cases} \tag{5}$$

where K in $\mathbb{R}^n$ is a gain matrix. The matrix function $\mathcal{L} : \mathbb{R}_+ \to \mathbb{R}^{n\times n}$ is defined as $\mathcal{L}(t) = \mathrm{diag}(L(t), \dots, L(t)^n)$ where $L : \mathbb{R}_+ \to \mathbb{R}$ is given as a solution to the following system of continuous–discrete differential equations

$$\dot{L}(t) = a_2 L(t) M(t) c(\hat{x}(t), u(t)), \quad t \in [t_k, t_{k+1}) \tag{6a}$$

$$\dot{M}(t) = a_3 M(t) c(\hat{x}(t), u(t)), \quad t \in [t_k, t_{k+1}) \tag{6b}$$

$$L(t_{k+1}) = L(t_{k+1}^-)(1 - a_1\alpha) + a_1\alpha \tag{6c}$$

$$M(t_{k+1}) = 1, \tag{6d}$$

initiated from $L(0) \geqslant 1$ and with $a_1\alpha < 1$. We have for all k,

$$y_k = Cx(t_k),$$

where the t_k's, k in $\mathbb{N}$ are given by the following relations,

$$t_0 = 0, \qquad t_{k+1} = t_k + \delta_k,$$
$$\delta_k = \min\{s \in \mathbb{R}_+ \mid sL((t_k+s)^-) = \alpha\}, \tag{7}$$

where α, a_1, a_2 and a_3 are positive real numbers to be chosen.

2.3. About the updating time period

To understand the motivation of this update law note that a first order approximation gives

$$L(t_{k+1}^-) = L(t_k) + a_2 L(t_k) c(\hat{x}(t_k), u(t_k))\delta_k + o(\delta_k).$$

Hence, taking into account that $\alpha = \delta_k L(t_{k+1}^-)$, it yields,

$$\frac{L(t_{k+1}) - L(t_k)}{\delta_k} = L(t_k)\left[a_1(1 - L(t_k)) + a_2 c(\hat{x}(t_k), u(t_k))\right] + o(1).$$

We recognize here the same update law structure than the one introduced in Praly (2003, equation (24)) which was motivated by a Riccati equation.

The sampling time interval which depends on L is well defined as this is shown in the following proposition.

Proposition 1 (*Sequence $(\delta_k)_{k\in\mathbb{N}}$ Well Defined*)**.** *If u is in $\mathbb{L}^\infty(\mathbb{R}_+, \mathbb{R}^p)$ then there exists a positive real number $\delta_{\min}$ depending on the initial condition $L(0)$ such that for all k in $\mathbb{N}$ there exists δ_k such that $\delta_{\min} \leqslant \delta_k \leqslant \alpha$.*

[1] The solution $\hat{x}(\cdot)$ is a right-continuous function. Given a right-continuous function $\phi : \mathbb{R} \to \mathbb{R}^n$, the notation $\phi(t^-)$ stands for $\phi(t^-) = \lim_{h\to 0, h<0} \phi(t+h)$.