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Brief paper

New high order sufficient conditions for configuration tracking[☆]

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ABSTRACT

In this paper, we propose new conditions guaranteeing that the trajectories of a mechanical control system can track any curve on the configuration manifold. We focus on systems that can be represented as forced affine connection control systems and we generalize the sufficient conditions for tracking known in the literature. The new results are proved by a combination of averaging procedures by highly oscillating controls with the notion of kinematic reduction.

1. Introduction

New geometric techniques are used to generalize tracking conditions known in the literature (Barbero-Liñán & Sigalotti, 2010; Bullo & Lewis, 2005; Chambrion & Sigalotti, 2008). The tracking problem plays a key role in the performance of robots and mechanical systems such as submarines and hovercrafts in order to avoid obstacles, stay nearby a preplanned trajectory, etc. Mechanical control systems are control-affine systems on the tangent bundle of the configuration manifold Q. In order to simplify the motion planning tasks for these control systems, a useful tool has been introduced in the geometric control literature, namely, the notion of kinematic reduction. Such a procedure consists in identifying a control-linear system on Q whose trajectories mimic those of the mechanical system. This approach has been useful to describe controllability, planning properties (Bullo & Lewis, 2005) and optimality (Barbero-Liñán & Munoz-Lecanda, 2010) of mechanical systems. However, as described in Bullo and Lewis (2005), kinematic reduction is not always possible, some conditions related to the symmetric closure of the control vector fields of both systems under study must be satisfied. In our previous work (Barbero-Liñán & Sigalotti, 2010) we proposed two extensions of the first-order sufficient conditions for tracking proposed in Bullo and Lewis (2005), each of them based on the construction of a family of 'compatible' vector fields, one family being of finite and the other one of infinite cardinality. Related constructions to generate admissible directions for tracking have been proposed in Bressan and Wang (2009); Martínez and Cortés (2003) (see also Agrachev & Sarychev, 2005, 2006). Our goal here is to obtain more general sufficient conditions for tracking, combining our previous results with the notion of kinematic reduction. More precisely, our aim is to identify conditions under which it is possible to associate with a mechanical system a kinematic reduction whose controlled vector fields are compatible with tracking in the sense of Barbero-Liñán and Sigalotti (2010). Trackability of the mechanical system will then follow from controllability of the kinematic reduction. The proposed approach applies directly to families of compatible vectors fields of finite cardinality (see Theorem 14). The infinite cardinality case requires some intermediate technical result. In particular, we are lead to establish a relationship between families of vector fields defined pointwise and sets of sections of the tangent bundle, in analogy to the classical Malgrange theorem (Malgrange, 1967). Based on such a pointwise characterization of infinite families of compatible vector fields, we obtain new sufficient conditions for tracking extending the results in Barbero-Liñán and Sigalotti (2010) (see Theorem 15). The newly obtained conditions are used to complete

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the analysis of the control properties of an underwater vehicle initiated in Chambrion and Sigalotti (2008), proving its trackability even in the most symmetric case (see Section 4.4 for details).

2. Notation and preliminaries

Denote by $\mathbb{N}$ the set of positive natural numbers and fix $n \in \mathbb{N}$. From now on, Q is a n-dimensional smooth manifold and $\mathfrak{X}(Q)$ denotes the set of smooth vector fields on Q. All vector fields are considered smooth as functions on Q, unless otherwise stated. Let $\tau_Q: TQ \to Q$ be the canonical projection. We denote by I a compact interval of the type $[0, \tau]$, $\tau > 0$.

2.1. Affine connection control systems

Definition 1. An **affine connection** is a mapping

$$\begin{aligned} \nabla: \mathfrak{X}(Q) \times \mathfrak{X}(Q) &\longrightarrow \mathfrak{X}(Q) \\ (X, Y) &\longmapsto \nabla(X, Y) = \nabla_X Y, \end{aligned}$$

satisfying the following properties: (1) ∇ is $\mathbb{R}$-linear in X and in Y; (2) $\nabla_{fX} Y = f\nabla_X Y$ for every $f \in \mathcal{C}^\infty(Q)$; (3) $\nabla_X fY = f\nabla_X Y + (Xf)\, Y$, for every $f \in \mathcal{C}^\infty(Q)$. (Here Xf denotes the derivative of f in the direction X.)

The mapping $\nabla_X Y$ is called the *covariant derivative of Y with respect to X*.

Definition 2. A **forced affine connection control system (FACCS)** is a control mechanical system given by $\Sigma = (Q, \nabla, Y, \mathscr{Y})$ where: Q is a smooth n-dimensional manifold called the *configuration manifold*, $Y : \mathbb{R} \times TQ \to TQ$ is smooth, affine with respect to the velocities, and such that $(\tau_Q \circ Y)(t, \cdot) = \tau_Q$ for every t, $\mathscr{Y}$ is a finite set $\{Y_1, \ldots, Y_k\}$ of control vector fields on Q. A trajectory $\gamma: I \subset \mathbb{R} \to Q$ is **admissible for Σ** if $\dot\gamma: I \to TQ$ is absolutely continuous and there exists a measurable and bounded control $u: I \to \mathbb{R}^k$ such that the dynamical equations of the control system Σ

$$\nabla_{\dot\gamma(t)} \dot\gamma(t) = Y(t, \dot\gamma(t)) + \sum_{a=1}^{k} u_a(t) Y_a(\gamma(t)), \tag{1}$$

are fulfilled (for almost every $t \in I$).

The vector field Y includes all the non-controlled external forces; e.g., the potential and the non-potential forces. The assumption that Y is affine with respect to the velocities means that, for every $q \in Q$ and $t \in \mathbb{R}$, the map $T_qQ \ni v \mapsto Y(t, v) \in T_qQ$ is affine.

3. Tracking problem

We consider here the problem arising when one tries to follow a particular trajectory on the configuration manifold, called *reference* or *target* trajectory, which is in general not a solution of the FACCS considered. A trajectory is successfully tracked if there exist solutions to the FACCS that approximate it arbitrarily well. Consider in what follows any distance $\mathrm{d}: Q \times Q \to \mathbb{R}$ on Q whose corresponding metric topology coincides with the topology on Q.

Definition 3. A curve $\gamma: I \to Q$ of class $\mathcal{C}^1$ is **trackable for the FACCS** Σ if, for every strictly positive tolerance ϵ, there exist a control $u^\epsilon \in L^\infty(I, \mathbb{R}^k)$ and a solution $\xi^\epsilon: I \to Q$ to Σ corresponding to u^ϵ such that $\xi^\epsilon(0) = \gamma(0)$ and $\mathrm{d}(\gamma(t), \xi^\epsilon(t)) < \epsilon$ for every $t \in I$. The trajectory is said to be **strongly trackable for** Σ if, in addition to the above requirements, for every $\epsilon > 0$ the approximating trajectory ξ^ϵ may be found also satisfying $\dot\xi^\epsilon(0) = \dot\gamma(0)$. A control system Σ satisfies the **configuration tracking property (CTP)** (respectively, the **strong configuration tracking property (SCTP)**) if every curve on Q of class $\mathcal{C}^1$ is trackable (respectively, strongly trackable) for Σ.

Remark 4. Since any $\mathcal{C}^1$ curve can be uniformly approximated, with arbitrary precision, by a smooth curve having the same tangent vector at its initial point, then Σ satisfies the CTP (respectively, the SCTP) if and only if every curve on Q of class $\mathcal{C}^\infty$ is trackable (respectively, strongly trackable) for Σ.

3.1. Tracking results for control-linear systems

A *control-linear system* (also called *driftless kinematic system*) on Q is a pair $(Q, \mathscr{X})$ where $\mathscr{X}$ is a finite subset $\{X_1, \ldots, X_m\}$ of $\mathfrak{X}(Q)$, identified with the control system

$$\dot\gamma(t) = \sum_{a=1}^{m} u_a(t) X_a(\gamma(t)), \quad \gamma(t) \in Q,$$

where $u_1, \ldots, u_m$ are L^∞ real-valued functions.

Proposition 5 (*See Liu, 1997; Sussmann & Liu, 1991*)**.** *Let $X_1, \ldots, X_m$ be smooth vector fields on Q and take $\kappa \in \mathbb{N}$. Let $\{X_1, \ldots, X_s\}$ be the set of all Lie brackets of the vector fields $X_1, \ldots, X_m$ of length less than or equal to κ. Assume that $\gamma : I \to Q$ is a $\mathcal{C}^\infty$ curve such that $\dot\gamma(t) = \sum_{a=1}^{s} w_a(t) X_a(\gamma(t))$, with $w : I \to \mathbb{R}^s$ smooth. Then, for every $\epsilon > 0$ there exists a solution γ_ϵ of the control-linear system $(Q, \{X_1, \ldots, X_m\})$ with smooth control $u_\epsilon : I \to \mathbb{R}^m$ and initial condition $\gamma_\epsilon(0) = \gamma(0)$ such that $d(\gamma(t), \gamma_\epsilon(t)) < \epsilon$ for every $t \in I$.*

From the above proposition we deduce the following result. (Similar arguments can be found in Jakubczyk (2002).)

Corollary 6. *If the Lie algebra $\mathrm{Lie}(X_1, \ldots, X_m)$ generated by $X_1, \ldots, X_m$ has constant rank on Q, then for every smooth curve $\gamma : I \to Q$ such that $\dot\gamma(t) \in \mathrm{Lie}_{\gamma(t)}(X_1, \ldots, X_m)$ for every $t \in I$ and for every $\epsilon > 0$ there exists a solution γ_ϵ of the control-linear system $(Q, \{X_1, \ldots, X_m\})$ with smooth control $u_\epsilon : I \to \mathbb{R}^m$ and initial condition $\gamma_\epsilon(0) = \gamma(0)$ such that $d(\gamma(t), \gamma_\epsilon(t)) < \epsilon$ for every $t \in I$.*

Proof. The proof works by covering the compact set $\gamma(I)$ by finitely many open sets $\Omega_1, \ldots, \Omega_K$ of Q such that for every $j = 1, \ldots, K$ there exists on Ω_j a basis of the distribution $\mathrm{Lie}(X_1, \ldots, X_m)$ made of Lie brackets of $X_1, \ldots, X_m$. Let κ be the maximum of the length of the brackets used to construct such bases and let $\{X_1, \ldots, X_s\}$ be the set of all Lie brackets of the vector fields $X_1, \ldots, X_m$ of length less than or equal to κ. Then $\dot\gamma(t) = \sum_{a=1}^{s} w_a(t) X_a(\gamma(t))$, with $w : I \to \mathbb{R}^s$ smooth, where smoothness follows from the fact that $\mathrm{Lie}(X_1, \ldots, X_m)$ has constant rank. We then conclude by Proposition 5. ■

3.2. Previous strong configuration tracking results

Conditions guaranteeing the SCTP have been obtained in Barbero-Liñán and Sigalotti (2010), generalizing previous results presented in Bullo and Lewis (2005) (in particular Theorem 12.26) and in Chambrion and Sigalotti (2008). We recall them here below in a version adapted to what follows. The main difference of these statements from the ones of Theorem 4.4 and Corollary 4.7 in Barbero-Liñán and Sigalotti (2010) is that here we focus on the strong configuration trackability of a given trajectory, instead of looking at the SCTP. The proof is however the same, since the proof proposed in Barbero-Liñán and Sigalotti (2010) is based on an argument where the target trajectory is also fixed.

We need to introduce the **symmetric product** in $\mathfrak{X}(Q)$ defined by $\langle X : Y\rangle = \nabla_X Y + \nabla_Y X$ for every $X, Y \in \mathfrak{X}(Q)$. For subsets A, B of $\mathfrak{X}(Q)$, $A - B = \{X - Y \mid X \in A,\ Y \in B\}$, $\mathrm{L}(A) = A \cap (-A)$, $\mathrm{co}(A)$ denotes the convex hull of A, and $\overline{A}$ is the closure of A in $\mathfrak{X}(Q)$ with respect to the topology of the uniform convergence on compact sets.