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Incomplete-profile potential games☆

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Abstract

The incomplete-profile normal form game (IPNFG), which contains several infeasible profiles, is considered. First, the dynamics of evolutionary IPNFG are presented. Then a method is provided to verify whether an IPNFG is potential. Certain properties of potential IPNFGs are revealed. Next, an algorithm is provided to search the feasible set to guarantee that the corresponding IPNFG is potential. In addition, the decomposition of an IPNFG is investigated. Finally, for an IPNFG with several feasible sets, an algorithm is proposed to find the one which makes the corresponding IPNFG closest to a potential game.

1. Introduction

In real world, there are some restrictions on people's decision-making under particular circumstances. To begin with, let us consider such a circumstance in a finite game: even though s_i, $i = 1, \dots, n$, are legal strategies (actions) for each players, respectively, the profile (joint action) $s = \prod_{i=1}^{n} s_i$ may not be feasible. A practical example is as follows:

There is a piece of chess board in Fig. 1. Now Player 1 can move the white rook from D1 to B1 (denoted by $s_1 \in S_1$), then Player 2 cannot move the black king from C2 to B3 (denoted by $s_2 \in S_2$), so the profile $s_1 \times s_2 \in S$ is not feasible, because C2 $\rightarrow$ B3 is a forbidden move for the black king by the rules of chess.

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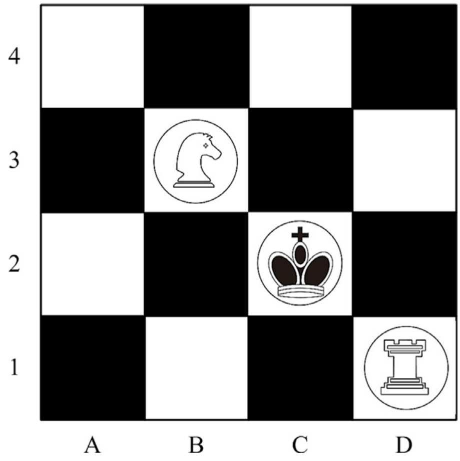

Fig. 1. Final stage of a game of chess.

Motivated by such kind of examples, a new class of games, named the incomplete-profile normal form games (IPNFGs), is proposed in this paper. The obvious difference between IPNFGs and the normal form games (NFGs) is that IPNFGs have some infeasible profiles. As shown in the previous example, it is easy to understand that the restrictions may come from somewhere, e.g., customs, rules, laws and regulations. Furthermore, when an evolutionary game (EG) is considered [1–3], its dynamics may depend on both strategy updating rule (SUR) and feasible set, which yields evolutionary IPNFGs.

The potential game was firstly proposed by Rosenthal [4]. After that, [5] systematically investigated potential games and presented several fundamental results such as its relation with finite improvement properties, its mixed extensions, etc. Since then, the theory of potential games has been applied to many engineering problems, for instance, to distributed power control and scheduling [6], to cooperative games [7], etc.

As pointed in [8]: “It is not easy, however, to verify whether a given game is a potential game”. Recently, the problem has been solved both theoretically and numerically [9,10]. In [9], Cheng proposed a linear system, called the potential equation (PE), and then proved that the game is potential if and only if the PE has at least one solution. In addition, a formula is provided to calculate the potential function, and the updated formula is proposed in [10]. Further more, the potential decomposition is firstly proposed by Candogan et al. in [11]. The decomposition of weighted potential games is investigated in [12].

The main purpose of this paper is to investigate potential IPNFGs. We will focus on these problems: (i) When an IPNFG is potential? (ii) How to delete some profiles to make the rest of a non-potential game a potential IPNFG? (iii) How to decompose an IPNFG to get its potential projection? To answer (i), a necessary and sufficient condition for verifying potential IPNFGs is obtained. As for (ii), an algorithm is developed to search a minimum deleting set. Certain properties of potential IPNFGs are also revealed. To solve (iii), the potential subspace of IPNFGs is investigated and an orthogonal decomposition is also presented.

The fundamental tool used in our approach is the algebraic state space representation of finite games based on semi-tensor product (STP) of matrices. This technique was firstly proposed in [13]. STP of matrices is a generalization of conventional matrix product to two

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