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Structural control of single-input rank one bilinear systems ☆

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ABSTRACT

A bilinear dynamical system can be used to represent the model of a network in which the state obeys linear dynamics and the input is the edge weight of certain controlled edges in the network. We present algebraic and graph-theoretic conditions for the structural controllability of a class of bilinear systems with a single control where the input matrix is rank one. Subsequently, we use these conditions, given a system state graph, to develop an algorithm to design the location of controlled edges (the input matrix) such that the system is structurally controllable.

1. Introduction

Recent studies have employed concepts from structural control in order to bring control theoretic analysis to large-scale complex networks. The rapid rise of computational capabilities and access to data in recent years has led to modeling many important systems – from intracellular biochemical pathways to the redesigned smart power grid – as networks (Newman, 2010). Understanding fundamental control properties is a key requirement to systematically studying and, ultimately, influencing these important systems. Classic control techniques, however, do not scale well to provide a feasible assessment of these properties. Structural control has proven to be a useful tool towards this goal.

Structural controllability is a generalization of classic controllability in which systems are analyzed based only on their structure, i.e., the existence or absence of a direct effect of one state on the change of another, and not the exact rate at which the states influence each other. Structural control is, therefore, "parameter free" in the sense that the analysis holds for all parameter values, except for specific pathological cases. This type of control is well-suited to analyze network systems by providing simplifications to make methods tractable and a set of tools that do not depend on exact parameter values, because such values are rarely known for most networks.

Conditions for structural controllability rely on classical control results, therefore, the analysis of networks has been limited to the case of linear dynamical systems, modeled as networks (Liu, Slotine, & Barabasi, 2011; Ruths & Ruths, 2014). While this body of work has already been able to provide revealing insights that connect network structures, such as the degree distribution, to control properties, more realistic models of these systems would permit deeper and more relevant analysis. In a network modeled by linear control dynamics, input signals are applied exogenously to specific nodes in the network, the influence of which is then able to control the entire network. This mechanism of influencing a network is applicable, for example, in resource networks (pipeline networks, power grids, and supply chains) where volume is injected or removed at nodes to manage demand, or food web networks where species can be bred and released or culled to achieve a population size (Dunne, Williams, & Martinez, 2002).

More often, however, this model falls short of how influence is achieved in a network. In a road network, tolls can be imposed on certain roads to alleviate traffic at specific points in the network. Similarly, biochemical networks are typically not controlled through direct injection of a protein, but instead by administering a drug that effects the rate at which that protein is produced naturally by the body (Marinissen & Gutkind, 2001).

A linear control model represents top-down control of a network whereas a bilinear model represents incentive-driven

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control from within a network. Both schemes require global oversight, since controls are generated centrally, however, the top-down (linear) scheme effects the states of the system directly, and the incentive-driven (bilinear) scheme effects the states indirectly by throttling the natural interaction between two states.

In this work we study structural control of bilinear systems for the purpose of applying these methods to networks with a bilinear control structure. We cannot leverage the work by Boukhobza and Hamelin (2007) on structural observability of bilinear systems, however, because the nonlinearity of bilinear systems does not enter the observability criterion, making controllability a significantly harder problem. Because structural controllability results rely on classic control results, we have built this work on top of the most general algebraic bilinear controllability results, which are known for a class of bilinear systems which have a single control and such that the input matrix is rank one (Evans & Murthy, 1977; Goka, Tarn, & Zaborszky, 1973). One of the major contributions of this work is the collection of intuitive graphical conditions which will be more easily generalized to a broader class of bilinear systems. At the same time this class of systems is not without direct application. Most systems employing regulatory control schemes are driven by a single controlling source with a broadcasting (rank one) structure of interaction. For example, the Federal Reserve sets the national interest rate so as to achieve market stability in the network of banks and loaning agencies within the United States.

Most network systems are composed of similar agents (nodes), and through their interaction the system evolves. Unlike engineered systems, in which, for example, a pump and a valve in a pipeline system are clear actuation points, these network systems do not have pre-established points at which control should be applied. In the network setting we need to design the input connectivity given the structured system so that the system and input together are controllable, a relatively new type of problem we call control configuration design. Methods for control configuration design exist for linear structured systems and correspond to selecting the (fewest) nodes in the network to receive exogenous input (Murota, 2000). Control configuration design for bilinear systems is, to date, an open question and involves placing additional (in particular, the fewest) edges with controlled edge weights such that the overall system of fixed edges and controlled edges is controllable. In the context of social networks, for example, this would be equivalent to establishing, removing, strengthening, or weakening interconnections of trust/mistrust among people so that the opinion of the group as a whole can be influenced.

The contributions of this work are twofold: first, for discrete-time single-input rank one bilinear systems we develop equivalent algebraic and graph-theoretic results for checking structural controllability, and second, we design an efficient algorithm for control configuration design for this same class of bilinear systems. Preliminary versions of these results were published in Ghosh and Ruths (2014a,b).

2. Background

We consider single-input homogeneous (without a linear control term) bilinear systems such that the input matrix is rank one. The state equation of the system is given by

$$\mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + u(t)\mathbf{B}\mathbf{x}(t) \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ denotes the state of the system and $u(t) \in \mathbb{R}$ denotes the control input to the system at time $t \in \mathbb{N}_0$; $\mathbf{A} \in \mathbb{R}^{n\times n}$ and $\mathbf{B} \in \mathbb{R}^{n\times n}$ denote the state and input matrices, respectively. Because we consider input matrices of rank one, the matrix $\mathbf{B}$ can be written as $\mathbf{B} = \mathbf{c}\mathbf{h}^{\mathrm{T}}$ where $\mathbf{c}, \mathbf{h} \in \mathbb{R}^n$. Thus, an alternate description of the state equation is

$$\mathbf{x}(t+1) = \mathbf{A}\mathbf{x}(t) + u(t)\mathbf{c}\mathbf{h}^{\mathrm{T}}\mathbf{x}(t). \tag{2}$$

Although select results exist for seemingly broader classes of bilinear systems, namely for controllability of multi-input and inhomogeneous systems, all of these results put highly restrictive assumptions on the form of the system matrices and are thus less general and interpretable (Evans & Murthy, 1978; Hollis & Murthy, 1981; Tie, Cai, & Lin, 2011). Even though the rank one condition on $\mathbf{B}$ is restrictive, a number of important classes of systems satisfy this requirement. One such example is the class of bilinear strict-feedback systems which are a class of nonlinear strict feedback systems (Khalil, 2002). For example, a strict feedback system of order 3 can be described by the following set of equations

$$\begin{aligned}
x_1(t+1) &= f_1\left(x_1(t)\right) + \gamma_1 x_2(t),\\
x_2(t+1) &= f_2\left(x_1(t), x_2(t)\right) + \gamma_2 x_3(t),\\
x_3(t+1) &= f_3\left(x_1(t), x_2(t), x_3(t)\right) + g_3\left(x_1(t), x_2(t), x_3(t)\right) u(t),
\end{aligned}$$

where the functions $f_i(\cdot)$ (with $i = 1, 2, 3$) and $g_3(\cdot)$ are linear in their variables; $\gamma_1, \gamma_2 \neq 0$. The overall state equation of the system is then given by (1) where $\mathbf{B}$ has all rows, except the last one, as zero rows. Another application of a single-input discrete-time rank-one bilinear system can be found in the context of wavelength-division multiplexing (Ishio, Minowa, & Noshu, 1984). The network consists of three parts: a multiplexer (that works according to the sparsity of $\mathbf{h}$), an amplifier with gain $u(t)$ and a demultiplexer (which works according to the structure of $\mathbf{c}$). Such networks appear the long-haul transmission where the sensors and actuators are far from each other and there is a bandwidth constraint of transmission and reception of data.

2.1. Structured systems

The notion of structured systems was introduced so that system properties could be evaluated and studied for systems that had a particular structure, regardless of the exact parameter values.

Mathematically the *structure* of structured systems is captured by matrix entries that are either fixed at zero (i.e., two states are known to have no direct interaction) or allowed to vary independently (i.e., the rate of the interaction between two states is given by an independent parameter). Therefore, in the structured version of (2) the structured matrices $\mathbf{A}$, $\mathbf{c}$, and $\mathbf{h}$ have entries that are either identically zero (denoted simply as 0) or free, able to take on any real number (denoted by λ_i or simply by $*$). An example of such a structured system is

$$\mathbf{x}(t+1) = \underbrace{\begin{bmatrix} 0 & \lambda_1 \\ \lambda_2 & 0 \end{bmatrix}}_{\mathbf{A}} \mathbf{x}(t) + u(t) \underbrace{\begin{bmatrix} 0 \\ \lambda_3 \end{bmatrix}}_{\mathbf{c}} \underbrace{\begin{bmatrix} \lambda_4 & \lambda_5 \end{bmatrix}}_{\mathbf{h}^{\mathrm{T}}} \mathbf{x}(t),$$

where $\lambda_i \in \mathbb{R}$ for $i \in \{1, \ldots, 5\}$ is an independent parameter. We study the properties of these systems in a generic sense; i.e., the properties under consideration must hold for almost every choice of these free parameters. We will define this notion in terms of polynomials and algebraic varieties.

An algebraic variety is the zero set of a finite set of polynomials. An algebraic variety $V \subset \mathbb{R}^N$ is called a proper variety if $V \neq \mathbb{R}^N$ and nontrivial if $V \neq \emptyset$. A proper variety is one of the standard sets known to have Lebesgue measure zero (Polderman & Willems, 1998)

Definition 1. A property (e.g., controllability) is said to hold generically for a structured system if the set of values of the free parameters for which the property does not hold forms a proper algebraic variety.