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Interval quadratic programming for day-ahead dispatch of uncertain predicted demand[☆]

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ABSTRACT

In this paper, we propose an interval quadratic programming method for the day-ahead scheduling of power generation and battery charge cycles, where the prediction uncertainty of power consumption and photovoltaic power generation is described as a parameter vector lying in an interval box. The interval quadratic programming is formulated as the problem of finding the tightest box, i.e., interval hull, that encloses the image of a function of the minimizer in parametric quadratic programming. To solve this problem in a computationally efficient manner, we take a novel approach based on a monotonicity analysis of the minimizer in the parametric quadratic programming. In particular, giving a tractable parameterization of the minimizer on the basis of the Karush–Kuhn–Tucker condition, we show that the monotonicity analysis with respect to the parameter vector can be relaxed to the sign pattern analysis of an oblique projection matrix. The monotonicity of the minimizer is found to be essential in the day-ahead dispatch problem, where uncertain predicted demand, described by a parameter vector, is dispatched to power generation and battery charge cycles while the economic cost is minimized. Finally, we verify the efficiency of the proposed method numerically, using experimental and predicted data for power consumption and photovoltaic power generation.

1. Introduction

Reductions in greenhouse gas emissions are recognized as a global goal, with renewable energy sources such as photovoltaic (PV) and wind power expected to contribute to an efficient solution. For example, large-scale penetration of PV power generators into Japanese houses is expected by 2030. In this situation, the total amount of PV power generation can cover approximately 50% of the peak power consumption, as well as 10% of the entire energy consumption (Masuta & Yokoyama, 2012).

With this background, we need to manage a power system that involves the traditional power generators as well as PV power generators and storage batteries while maintaining the balance among the amounts of power generation, demand, and battery charging power. To improve the economic efficiency of power system management, the day-ahead schedules of power generation and battery charge cycles can be based on day-ahead predictions of power consumption and PV power generation (Hanna, Kleissl, Nottrott, & Ferry, 2014; Li, Ding, Li, & Dan, 2012; Riffonneau, Bacha, Barruel, & Ploix, 2011). In the following, we use the term "demand" to

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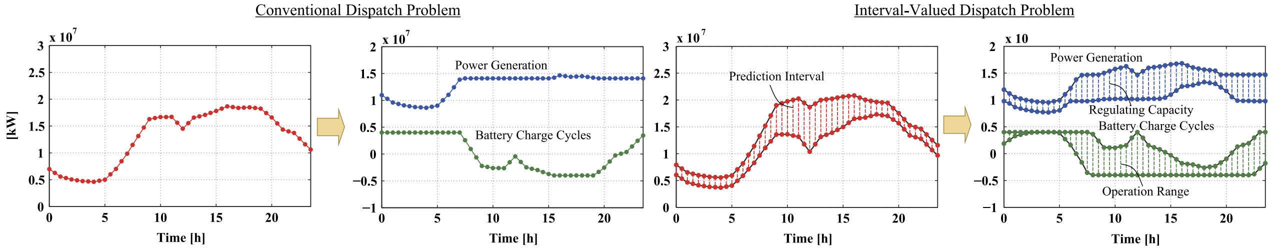

(a) Demand Prediction Profile.

(b) Profiles of Power Generation and Battery Charge Cycles.

(c) Demand Prediction Interval.

(d) Intervals of Power Generation and Battery Charge Cycles.

Fig. 1. Dispatch problems arising in scheduling of power generation and battery charge cycles.

represent the difference between the total consumer demand and the amount of PV power generation. As shown in Fig. 1(a) and (b), the minimization of economic cost, such as the fuel cost of generators and the deterioration of storage batteries, can be regarded as a dispatch problem in which the predicted demand profile is divided into that for power generation and for battery charge power.

However, sharp fluctuations in PV power generation and idiosyncratic power consumption make the exact prediction of demand profiles difficult. In view of this, a number of prediction methods for renewable energy generation have been developed in the literature. For example, Appel, Gilliam, Davis, Zubrow, and Howard (2011) and Bofinger and Heilscher (2006) utilize weather forecasts and numerical simulations of meteorological phenomena to produce prediction profiles of renewable energy. On the other hand, as a relatively new approach, there are several methods for determining confidence intervals for renewable energy prediction (Petiau, 2009; Seppälä, 1996), representing an interval in which prediction profiles are contained with a certain probability (e.g., 95%). To comply with such an interval-valued prediction method, we model the uncertain demand prediction as a set of all demand profiles that lie in an *interval box*. In particular, we use a confidence interval for PV power generation prediction produced by the method proposed in Silva Fonseca, Oozeki, Ohtake, and Ogimoto (2014). In this method, a prediction model based on support vector regression produces an interval with a predetermined confidence level from meteorological data (JMBSC, 2015), involving ambient temperature, humidity, and cloudiness. Its performance analysis is performed in Silva Fonseca et al. (2014) showing that the Laplacian distribution is better than the Gaussian distribution to approximate the expected variation of the prediction error coverage with relatively high confidence levels.

The day-ahead scheduling based on this type of prediction leads to an interval-valued dispatch problem, as shown in Fig. 1(c) and (d). Note that our demand dispatch is clearly different from the standard one (Dicorato, Forte, Pisani, & Trovato, 2012; Riffonneau et al., 2011; Teng, Luan, Lee, & Huang, 2013) in the sense that we deal with the prediction uncertainty as an interval-valued parameter. The resultant interval of power generation profiles corresponds to the regulating capacity of power generators that is sufficient to tolerate a given amount of prediction uncertainty.

In this paper, we perform the interval-valued dispatch via *interval quadratic programming*. The interval quadratic programming is formulated as the problem of finding the tightest box, i.e., interval hull, that encloses the image of an output function consisting of the minimizer of a parametric quadratic program, where the parameter vector lies in an interval box. The problem can also be regarded as a type of reachability analysis problems (Althoff, 2010; Le Guernic & Girard, 2010; Ramdani, Meslem, & Candau, 2009) in the context of parametric quadratic programming. This is because we aim at capturing the image of a mapping function defined over an interval box. It should be emphasized that such a problem is not necessarily easy to solve, because the exact interval hull is not obtained by calculating minimizers for a finite number of grid points in the parameter space.

To overcome this difficulty, we take a novel approach based on a monotonicity analysis to calculate the interval hull of interest. This novel approach has the advantage that we can calculate the exact interval hull in a finite number of operations. More specifically, it is theoretically guaranteed that the minimizer with respect to some extreme points of the interval box gives the exact interval hull. This approach has the potential to handle large-scale problems with high-dimensional decision variables and parameter space.

To clarify our theoretical contribution for interval-valued optimization, some references on interval analysis theory are in order. In fact, most studies on interval analysis focus on global optimization, i.e., finding the global extremum of a multi-modal multi-variable function (Ichida, 1996; Mazhoud, Hadj-Hamou, Bigeon, & Remy, 2012), or on constraint satisfaction problems, i.e., covering a set of feasible solutions complying with equality and inequality constraints (Jaulin, 2001). Towards these objectives, a constraint propagation technique is commonly used in conjunction with branch-and-bound algorithms (Hansen & Walster, 2003; Jaulin, 2001). Even though the application of these methods can address interval quadratic programming, this requires the direct computation and partition of the interval of real numbers, thereby incurring large computation loads. Moreover, overestimation often causes the resultant solution to be conservative.

Algorithms for parametric quadratic programming have been developed for model predictive control (Bemporad, Morari, Dua, & Pistikopoulos, 2002; TøNdel, Johansen, & Bemporad, 2003). Even though these deal with a similar type of parametric quadratic programming, their focus is mainly on partitioning the parameter space with respect to the index sets of active inequality constraints. In this sense, their objective is clearly different from ours because we aim to capture the image of minimizers over the parameter space. Furthermore, the application of the algorithms in Bemporad et al. (2002), TøNdel et al. (2003) to large-scale problems is not realistic, because their reliance on the iterative enhancement of the search space often entails a considerable computation load. To the best of our knowledge, there is no computationally efficient method to handle general interval-valued optimization. It should be noted that, even though the papers (Hladík, 2011; Li & Tian, 2008) handle nonlinear programming problems with interval-valued data, the problem formulation is totally different from that in this paper because they focus on calculating the optimal value bounds of an objective function.

In addition, we reference some related studies to clarify our contribution from the viewpoint of its application. Several studies on the scheduling of power generation and battery charge cycles have considered the prediction uncertainty using stochastic optimization (Abbey & Joós, 2009; Aihara, Yokoyama, Nomiyama, & Kosugi, 2010; Arun, Banerjee, & Bandyopadhyay, 2009; Bu, Yu, & Liu, 2011; Sturt & Strbac, 2012; Yang, Yu,