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Design of stable parallel feedforward compensator and its application to synchronization problem[☆]

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ABSTRACT

This paper addresses the design problem of a stable parallel feedforward compensator V for a given SISO LTI plant P (possibly being of non-minimum phase and/or having relative degree greater than one). The objective of the problem is that their parallel interconnection P + V becomes minimum phase having relative degree one. Based on the classical results of simultaneous stabilization, a necessary and sufficient condition for solving the problem is presented as well as a design procedure for constructing such a compensator. The proposed feedforward compensator allows the control system to have the three useful features: (1) the ability that assigns the zeros of P + V to a region of complex numbers having arbitrary negative real parts, (2) infinite gain margin property of P+V controlled by a static output feedback, and (3) block diagonal structure of P+V. These features are extensively exploited in the synchronization problem of multi-agent systems to achieve arbitrary fast convergence rate and to have the synchronized trajectory independent of the initial conditions and parameters of the involved dynamic controllers.

1. Introduction

A class of systems being of minimum phase and having relative degree one is frequently encountered, and is dealt with in depth in the literature because it is closely related with passive systems (Byrnes, Isidori, & Willems, 1991; Sepulchre, Janković, & Kokotović, 1997) and a system belonging to this class can be stabilized through a simple static high-gain output feedback (Byrnes et al., 1991; Isidori, 1995). Although the high-gain stabilization schemes can be extended to systems having higher relative degree as long as they remain as minimum phase (Teel & Praly, 1995), the output feedback control of non-minimum phase systems still suffers from its intrinsic limitations.

A possible approach to overcome these difficulties is to find a parallel feedforward compensator (PFC) V such that the parallel interconnection P + V of the plant P and the compensator V, shown in the shaded region of Fig. 1(a), has desired properties (e.g., passivity, minimum phaseness, and/or relative degree 1) when the signal $y = y_p + y_v$ is viewed as a new output. (Therefore, this idea belongs to the category of *output redefinition* methods.) If this task is successfully done, then, relying on the obtained properties of P + V, one may design an output feedback controller Q for P + V to achieve the objectives of the original problem with relative ease. These feedforward and feedback controllers are actually implemented in a feedback form like in Fig. 1(b), i.e., V becomes a part of a *feedback* controller.

In this direction, Bar-Kana (1987) has used a PFC to make P + V almost strictly positive real (ASPR), which results in an implementable simple adaptive controller (Sobel, Kaufman, & Mabius, 1982) when the plant itself does not satisfy the positivity condition. In particular, he showed in Bar-Kana (1986) that if $P(s)$, the transfer function of the system P, can be stabilized by a biproper output feedback controller $V^{-1}(s)$, then $P(s)+V(s)$ becomes ASPR. Following the approach and results of Bar-Kana (1986, 1987), a robust design problem of such PFCs for uncertain plants has been addressed in, e.g., Deng, Iwai, and Mizumoto (1999), Iwai and

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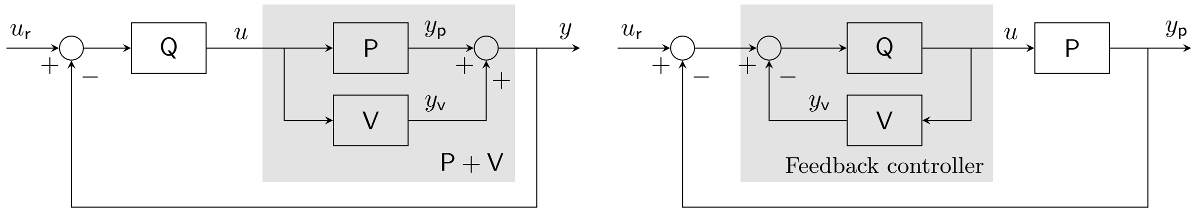

(a) Feedforward and feedback control. (b) An equivalent feedback implementation.

Fig. 1. Feedforward control scheme and its equivalent representation in feedback control.

Mizumoto (1992, 1994) and Iwai, Mizumoto, and Deng (1994) with constructive design methods given. A common restriction of them is $P(s)$ has to be of minimum phase uniformly in the uncertainties. Several passification methods of non-passive systems via suitable compensation, including PFC, were also introduced by Kelkar and Joshi (1997).

On the other hand, state space design approaches of PFC have been reported in, e.g., Isidori and Marconi (2008), Misra and Patel (1988), Patel and Misra (1992) and Son, Shim, Jo, and Seo (2003); Son, Shim, Park, and Seo (2002). In Misra and Patel (1988) and Patel and Misra (1992), the design methods of PFC V were provided in order that P + V is of minimum phase and has relative degree zero. Son et al. (2003, 2002) have considered the problem of designing a PFC that achieves relative degree 1 and minimum phaseness of the interconnected system. In particular, they showed the problem is solvable if there exists a static output feedback controller that stabilizes a certain system derived from the plant P. Requiring the existence of a static stabilizer is a consequence of the PFC to be input-dimensional (i.e., the dimension of the PFC is the same as that of the plant input). In Isidori and Marconi (2008), the design problem of a PFC for a class of nonlinear plants was addressed and it was shown that the output feedback stabilization problem of a nonlinear plant P is solved by means of the proposed PFC V and the static high-gain output feedback of P + V. The PFC V in Isidori and Marconi (2008) was constructed from the prior knowledge of a controller that possesses a certain structural property and stabilizes the auxiliary system (again derived from P).

However, while those references present different sufficient conditions for the existence of the PFC, a complete necessary and sufficient characterization and a systematic design method are still lacking. Motivated by this fact, we study a necessary and sufficient condition for the existence of *stable* PFC and its constructive design procedure. It will be seen in Section 3 that, by imposing the stability of PFC itself, the problem can be converted into the classical simultaneous stabilization problem and therefore, a few off-the-shelf design methods become applicable to the PFC problem. The search for PFC within the stable systems could be a restriction, but it also provides with some benefits. One particular application, where the structure of P + V and stability of V play important roles, is the synchronization problem dealt with in Section 4. Synchronization problem has its origin at the asymptotic convergence of each first-order dynamic agent to their average of initial conditions (Jadbabaie, Lin, & Morse, 2003; Ren, Beard, & Atkins, 2007). It has been extended to higher order dynamic agents, but instead some dynamic controller is introduced into each agent. As a result, the synchronized trajectory is not purely an average of each agent, but the initial conditions of the controllers now take part in the average (see, e.g., Kim, Shim, Back, & Seo, 2013; Li, Duan, & Chen, 2011; Li, Duan, Chen, & Huang, 2010; Scardovi & Sepulchre, 2009 and Seo, Shim, & Back, 2009). On the other hand, the proposed design of synchronizing controller overcomes this drawback and the problem reverts into its original philosophy. Moreover, the proposed design of PFC allows arbitrarily fast zero assignment (that has not been addressed in the previous results, e.g., Bar-Kana, 1986; Deng et al., 1999; Iwai & Mizumoto, 1994 and Son et al., 2002), which in turn yields fast synchronizing rate to the average. This feature also eliminates the intrinsic limitation of Seo et al. (2009), in which the synchronizing rate is usually very slow because of its low-gain based design. Finally, the PFC admits infinite gain margin property of P + V,[2] which enables the design of fully distributed synchronizing controllers (Li, Ren, Liu, & Fu, 2013; Li, Ren, Liu, & Xie, 2013) that do not use the information on the interconnection structure of network. We mention that the topics covered in this paper have their origins in Kim (2011) and Kim, Kim, Back, Shim, and Seo (2011) in part.

Notation. For $a \in \mathbb{R}$, $\mathbb{C}_{\geq a}$ denotes the set of complex numbers whose real parts are greater than or equal to a, namely, $\mathbb{C}_{\geq a} := \{s \in \mathbb{C} : \mathrm{Re}(s) \geq a\}$. In addition, $\mathbb{R}_{\geq a} := \{s \in \mathbb{R} : s \geq a\}$ and $\mathbb{R}^{\infty}_{\geq a} := \mathbb{R}_{\geq a} \cup \{s = +\infty\}$. The sets $\mathbb{R}_{>a}$, $\mathbb{C}_{<a}$, and $\mathbb{C}^{\infty}_{\geq a}$ are defined analogously. $\mathcal{S}_{<a}$ denotes the set of proper rational functions whose poles are all in $\mathbb{C}_{<a}$. A controller $C(s)$ is said to stabilize a plant $P(s)$ (Vidyasagar, 1985) if the closed-loop system $\mathcal{H}(P(s), C(s))$ is stable in the sense that

$$\mathcal{H}(P(s), C(s)) := \begin{bmatrix} 1/\Delta(s) & -P(s)/\Delta(s) \\ C(s)/\Delta(s) & 1/\Delta(s) \end{bmatrix} \in \mathcal{M}(\mathcal{S}_{<0}),$$

where $\Delta(s) := 1 + P(s)C(s)$ and $\mathcal{M}(\mathcal{S}_{<a})$ is the set of 2×2 matrices whose elements are in $\mathcal{S}_{<a}$. Two plants $P_0(s)$ and $P_1(s)$ are *simultaneously stabilizable* if there is a common controller $C(s)$ that stabilizes both of the plants, i.e., $\mathcal{H}(P_i(s), C(s)) \in \mathcal{M}(\mathcal{S}_{<0})$ for $i = 0, 1$. $[x; y]$ stands for the stack of two vectors (or matrices of compatible dimensions) x and y. The symbols $\top$ and $\otimes$ denote transpose and Kronecker product, respectively.

2. Design problem of stable PFC

Consider a SISO LTI plant P given by

$$\mathsf{P} : \begin{cases} \dot{p} = A_p p + B_p u, & p \in \mathbb{R}^n, \ u \in \mathbb{R}, \\ y_p = C_p p, & y_p \in \mathbb{R}, \end{cases} \tag{1}$$

where (A_p, B_p, C_p) is minimal, i.e., controllable and observable.

In this paper, whenever we call a 'stable parallel feedforward compensator (PFC)', we mean a dynamical system of the form

$$\mathsf{V} : \begin{cases} \dot{v} = A_v v + B_v u, & v \in \mathbb{R}^m, \\ y_v = C_v v, & y_v \in \mathbb{R} \end{cases} \tag{2}$$

such that the parallel interconnection P + V (see Fig. 1)

$$\begin{aligned} \dot{x} &= Ax + Bu := \begin{bmatrix} A_p & 0 \\ 0 & A_v \end{bmatrix} x + \begin{bmatrix} B_p \\ B_v \end{bmatrix} u, \\ y &= Cx := \begin{bmatrix} C_p & C_v \end{bmatrix} x \end{aligned} \tag{3}$$

[2] By "infinite gain margin property of P + V", we mean P + V controlled by a static output feedback $\pm u = -ky$ remains stable for any $k > k^\star$ and some $k^\star > 0$. (The sign in front of u depends on the context.)