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Learning to cooperate: Networks of formation agents with switching topologies[☆]

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ABSTRACT

Motivated by the prototypical problem of a marching band, this paper studies a class of multi-agent formation problems characterized by two features: (i) the agents attempt to complete a finite duration, coordinated formation task with high precision by repeating the task and (ii) the feedback mechanism by which the agents control their motions is based on relative differences between nearest neighbors, but the underlying graph topology can vary both during a repetition and from one repetition to the next. Adopting the framework of iterative learning control leads to the notion of multi-agent networks with switching topologies along two directions: a finite time axis and an infinite iteration axis. For such systems, we present distributed algorithms using nearest neighbor information whose exponential convergence can be demonstrated. It is shown that as the number of repetition increases, the relative formation between agents approaches the desired formation exponentially fast if and only if at each time step, the union of the interaction graphs has a spanning tree frequently enough along the iteration axis. That is, the agents can "learn to cooperate." The remarkable point of this result is that it is not necessary to have a spanning tree at any specific time step or iteration in order for the system to converge. Two examples are given to illustrate the ideas, including a general example, where through iteration the agents can form a desired formation, and a special case of it, where an additional agent specifies a reference to regulate the formation shape simultaneously.

1. Introduction

1.1. A motivating example: the "marching band" problem

Consider the example of a marching band, which consists of a group of performers that "march", or walk, through several formations while playing musical instruments. The goal of any individual performer is to "be at the right place at the right time", relative to the other performers. Typically, such a group achieves its objectives through practice. Further, though each performer is given an individual trajectory, it is well known to those who have participated in such an activity that to get it right you must consider the relative distance to your neighbors throughout the performance. Each of these points is reinforced in the on-line wikiHow article *How to Practice Marching Band Formations*, which notes, "Keep your eyes off the ground. It isn't going anywhere! If you rely on the ground to know where you are supposed to go, you will be learning nothing and will regret it later on, such as during performances … Rely on the distance you are away from others to determine if you are in the correct position … Repetition is key. Try practicing your formations …" (http://www.wikihow.com/Practice-Marching-Band-Formations).

In a marching band, during each practice repetition every performer observes the relative distance from his/her nearest neighbors and consciously plans changes in their motion for the next practice attempt (usually a neighbor is another performer that is within line-of-sight, but possibly also within sphere-of-sound).

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As the group continues to practice, the correct motion can be learned and stored in the performers' memories such that the formation accuracy is iteratively improved. This actually describes a "learning to cooperate" picture of groups of performers whose implementation strategies are generated through repetition and nearest neighbor communications.

If we consider the marching band from the perspective of a classical multi-agent network (see Bullo, Cortés, & Martínez, 2009, Cao, Yu, Ren, & Chen, 2013, Mesbahi & Egerstedt, 2010, Olfati-Saber, Fax, & Murray, 2007 and Ren, Beard, & Atkins, 2007), we can view each performer as an individual agent and his/her position as the agent's dynamically changing state. It is natural in that context to view the communication between two performers as carried out through vision-based or sound-based observations of the difference in location of each performer to the other. If we then add to this characterization the repetition that takes place as a marching band practices its formation, we are thus led to consider the notion of iterative learning control in the context of multi-agent consensus.

Iterative learning control, or ILC, is a well-established technique for improving the performance of a controlled dynamic system that is operated to repeatedly follow the same finite-length desired trajectory over-and-over. ILC techniques exist for a variety of problem scenarios ranging from linear to non-linear plants, both SISO and MIMO, as well for problems involving various forms of uncertainty. See Ahn, Chen, and Moore (2007), Bristow, Tharayil, and Alleyne (2006) and Xu (2011) and the references therein.

1.2. Related works

Interpreting a marching band as a multi-agent system that improves its performance through practice leads to the phrase "learning to cooperate", whereby a group of agents learns to execute a cooperative task via repetition of that task. In the literature and applications, multi-agent systems commonly consist of a group of mobile agents that are required to perform a cooperative task for the group (see, e.g., Bullo et al., 2009, Cao et al., 2013, Mesbahi & Egerstedt, 2010, Olfati-Saber et al., 2007 and Ren et al., 2007 and references therein). Consensus and formation control are two of the most common problems considered in the literature (see, e.g., Abdessameud & Tayebi, 2013, Jadbabaie, Lin, & Morse, 2003, Lin, Francis, & Maggiore, 2005, Moreau, 2005, Olshevsky & Tsitsiklis, 2009, Ren & Beard, 2005, and Schenato & Fiorentin, 2011). A number of problems in multi-agent cooperation include the idea of repetition of system operation. Some examples, similar to the marching band problem described above, include synchronized swimming, soldiers marching in formation, and aerial flight displays involving multiple aircraft. In addition, high precision control is often required by many cooperative tasks that exhibit repetition. Because it is well known that ILC methods have the ability to refine arbitrary high precision tracking performance, noted in the surveys (Ahn et al., 2007; Bristow et al., 2006; Xu, 2011), a number of papers have applied the ILC paradigm to cooperative problems requiring high precision. For example see Ahn, Moore, and Chen (2010) for trajectory keeping of satellites, Chen and Jia (2010) for formation control of robots, and Sun, Hou, and Li (2013) for trajectory tracking of trains. Other coordination learning algorithms for multi-agent systems can be found in, e.g., Ahn and Chen (2009), Li and Li (2014), Liu and Jia (2012), Meng, Jia, and Du (2013a,b, 2015); Meng, Jia, Du, and Yu (2012); Meng, Jia, Du, and Zhang (2014), Shi, He, Wang, and Zhou (2014) and Yang, Xu, Huang, and Tan (2014); Yang, Xu, and Yu (2013). It has been shown that cooperative tasks can be achieved with the arbitrary high precision even for multi-agent systems associated with generic directed graphs.

Although the literature shows that under various assumptions it is possible to achieve good results for coordination learning problems, these results are far from complete and there remain many open problems. In particular, it is worth noticing that in the marching band example and other problems in this class, the nearest neighbor topology by which relative distance is communicated between agents will change between formations. As such, we are led to the need to investigate convergence for coordination learning processes when they are subject to switching network topologies. Generally, we can consider that topology switching can occur not only relative to time step, as in the marching band example, but also relative to repetition. This possibility arises because, as we show below, the coordination learning process is effectively a plant with two independent variables: time step and learning repetition. When we introduce switching topologies in both dimensions, time and repetition, our problem becomes that of ILC for plants with both time- and repetition-dependent parameter variations. We will approach this as a class of convergence problems for two-dimensional (2-D) Roesser systems subject to both time- and repetition-dependent parameter variations, for which the equilibrium trajectory depends on the switching topologies and is not known explicitly. This is different from the stability problem of 2-D Roesser systems considered in the literature for classical ILC, e.g., Kurek and Zaremba (1993), Li, Ho, and Chow (2005), Meng, Jia, Du, and Yuan (2009) and Saab (1995), because of those parameter variations, which make the convergence and even the stability properties hard to analyze in general.

Although there have been some results addressing the coordination learning problem for multi-agent systems with switching topologies by considering different dynamics of agents in Liu and Jia (2012), Meng et al. (2013a,b, 2015, 2014) and Yang et al. (2013), these proposed results are subject to some drawbacks that can be further improved. In Meng et al. (2013a), the network topology is switching relative to time but invariant with iteration. The multi-agent systems can be considered under 2-D switching topologies in Liu and Jia (2012), Meng et al. (2015) and Yang et al. (2013), for which their associated interaction graphs are, however, required to have a spanning tree or be strongly connected at all time steps and all learning repetitions. In Meng et al. (2013b), only the convergence of multi-agent systems at the terminal time can be guaranteed under the ILC-motivated algorithms. Though the network topology condition in Meng et al. (2014) can tolerate 2-D switching topologies and also allows the interaction graphs to not contain spanning trees at any time step or any learning repetition, it requires that the union of interaction graphs is strongly connected frequently enough over iteration intervals of finite length. In addition, only sufficient convergence conditions can be developed for multi-agent systems under iteration-varying network topologies (Liu & Jia, 2012; Meng et al., 2013b, 2015, 2014; Yang et al., 2013), whereas necessary and sufficient convergence conditions can be obtained under iteration-invariant network topologies (Meng et al., 2013a, 2012). This is because, for example, an eigenvalue analysis is applicable under the iteration-invariant topology condition but not under the iteration-varying topology condition.

1.3. Our contributions

In this paper, we address the coordination learning problems for multi-agent formation systems under 2-D switching topologies and establish their convergence properties with an exponentially fast speed. Our results develop necessary and sufficient convergence theorems for multi-agent formation problems. We consider the 2-D (time and repetition) multi-agents systems over directed graphs with topologies switching in both dimensions. Using the nearest neighbor rule, we apply distributed algorithms for