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Necessary and sufficient conditions for asymptotic model matching of switching linear systems☆

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ABSTRACT

This paper investigates the problem of designing a feedback controller to force the response of a given plant to match asymptotically that of a prescribed model, in case both the plant and the model are switching linear systems. Matching has to be achieved, with stability of the feedback loop, for any initial conditions of the plant, the model and the controller (in case it is dynamic) and for any choice of the switching law. Solvability of the problem is characterized in terms of necessary and sufficient conditions that, in part, refer to the geometric structure of the difference system which compares the output of the model and of the plant. Stability is considered both for slow switching and for arbitrary switching. Proofs of the solvability conditions provide viable procedures for synthesizing the static or dynamic controllers that achieve the matching, respectively, when the state of the model is measurable and when it is not. Weaker sufficient conditions that can be practically checked by simple algorithmic procedures are provided, both in the general situation and under slightly restrictive hypotheses.

1. Introduction

The problem we consider in this paper consists in compensating a given plant Σ_P in such a way to stabilize it and to force its output to match asymptotically that of a given model Σ_M, for any arbitrary initial condition of the plant, of the model and of the compensator and for any admissible input, in case both the plant and the model are switching linear systems. Namely, each of them is a dynamical structure that consists of an indexed family $\Sigma = \{\Sigma_i\}_{i\in I}$, where I is a finite set of linear systems, called modes, having the same input, output and state space, and of a measurable, arbitrary signal $\sigma : \mathbb{R}^+ \to I$, that defines the switching from one mode to another (see, e.g., Liberzon, 2003; Lin & Antsaklis, 2009; Sun & Ge, 2005).

It is not difficult to understand that a necessary requirement for achieving asymptotic matching for any arbitrary initial condition and any admissible input is that the output of the compensated plant matches exactly (i.e. is equal to) the output of the model for any admissible input when, in particular, the initial conditions of the plant, of the compensator and of the model are all equal to zero. In those terms, namely in terms of exact matching of the forced responses (i.e. of the outputs that correspond to given inputs and to zero initial conditions), the problem was first considered in the early 1970s by Wolovich (1971) in the context of linear systems. Its theoretical and practical relevance is due to the fact that a large number of regulation and control problems (related, in particular, to model following and tracking, to servomechanism design, to model reference adaptive control) can be dealt with in terms of model matching. Basically, given a plant and a number of specifications that define the desired performances, the design of a controller that forces the plant to meet the specifications, if any exists, can be accomplished by taking a model that has the desired characteristics and by searching for a controller that forces the plant to match the model (see Ichikawa, 1985). It is evident, from this, the interest in finding conditions that characterize the solvability of a given model matching problem as well as in devising a viable synthesis procedure. This was recognized by many authors and it led to studying the model matching problem, using different approaches, in several contexts, including, besides that of linear systems, those of nonlinear systems, periodic systems, time-delay systems, 2D systems, time-varying systems. Among the wide literature on the subject, important contributions are found in Chu and Van Dooren (2006); Colaneri and Kučera

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(1997); Conte and Perdon (1995); Di Benedetto and Grizzle (1994); Di Benedetto and Isidori (1986); Hikita (1981); Kase, Mutoh, and Okuno (1998); Kristalny and Mirkin (2012); Levinson and Mirkin (2012); Malabre and Kucera (1984); Marinescu (2009); Marinescu and Bourles (2003); Marro and Zattoni (2005); Moog, Perdon, and Conte (1991); Moore and Silverman (1972); Morse (1973); Morse (1996); Paraskevopoulos (1978); Picard, Lafay, and Kucera (1998); Tzafestas and Paraskevopoulos (1976); Vardulakis and Karcanias (1985); Wang and Desoer (1972); Wolovich (1972); Xu and Hong (2013); Yamanaka, Ohmori, and Sano (1997) and Yang (2011). The development of methods for achieving model matching in the switching framework can add new practical design tools for dealing with regulation and control problems in this area.

The contribution of the present paper is in the same line of those just mentioned and it is summarized as follows. We state formally the asymptotic model matching problem with stability in the case in which, assuming that the state of the model is measurable, the matching can be achieved by using a static state feedback compensator (Stable Model Matching Problem with Static Feedback, or SMMPSF) and in the case in which, assuming that the state of the model is not measurable, a dynamic state feedback compensator is required (SMMP with Dynamic Feedback, or SMMPDF). Stability, as it is customary in the framework of switching systems (see Liberzon, 2003; Lin & Antsaklis, 2009), is considered separately for the case of slow switching, in which restrictions in terms of dwell time are imposed on the set of admissible switching signals, and for the case of arbitrary switching, in which no restrictions (except a technical one to avoid chattering behavior) are imposed on the set of admissible switching signals. Then, in Theorem 1 we give a complete characterization, in terms of necessary and sufficient conditions, of the solvability of the asymptotic model matching problem with stability for sufficiently slow switching, both in the case of measurable and of non measurable state of the model (see Remark 4 for a discussion about the minimum dwell time that assures asymptotic stability). These results are obtained under the very mild hypothesis that the modes of the plant are asymptotically stabilizable and those of the model are asymptotically stable. With more information on the plant and on the model, namely assuming that both are stable for (slow) switching with a given dwell time τ_α, in Theorem 2, we give a complete characterization, in terms of necessary and sufficient conditions, of the solvability of the SMMPSF and of the SMMPDF for (slow) switching with the same dwell time τ_α. Moreover, in that theorem, under the hypothesis that the plant and the model are stable for arbitrary switching, we also completely characterize the solvability of the SMMPSF for arbitrary switching and we give a sufficient condition for the solvability of the SMMPDF for arbitrary switching. It is important to remark that the solvability conditions of Theorems 1 and 2 are constructive, in the sense that they provide an algorithmic synthesis procedure for the controllers that achieve the matching.

To the best of the authors' knowledge, no other alternative, complete characterization of the solvability of the asymptotic matching problem in the general context of switching systems and no other general synthesis procedure of a solution in the switching context are available in the literature. The asymptotic model matching was considered in Balas, Bokor, and Szabó (2004) for a class of LPV systems whose dynamic, input and output matrices, in state space form, are affine combinations with coefficient functions $\rho_i(t)$ of a finite number of given constant matrices. A similar representation can be used for the dynamic, input and output matrices of switching systems, with coefficient functions that assume discontinuously only the values 0 and 1. Nevertheless, switching systems are structurally different from LPV systems as explained in Hespanha (2004) and, therefore, the results of that paper do not apply directly to the general case we consider here. Nor they can be extended to it, since in Balas et al. (2004) the solvability conditions and the synthesis procedure of the controller are derived assuming and exploiting the differentiability of the coefficient functions $\rho_i(t)$ (see Balas et al., 2004, Remark 1), the invertibility of the plant and the equality of the input matrices of all modes of the plant (see Balas et al., 2004, Equation (37)). None of the two latter restrictive assumptions is made in Theorems 1 and 2 and, moreover, differentiability of the coefficient functions in the recalled representation of switching systems does not hold. Also note that the asymptotic model matching problem we consider here is different from the matching problem studied in Jiang and Voulgaris (2009); Naghnaeian and Voulgaris (2012), which consists in compensating a switching plant in such a way to minimize the norm of the output difference system between the compensated plant and a given model over all possible switching signals. Those papers deal only with very restricted classes of switching systems (the so-called output or input–output switching systems), whose structure is totally different from that of the general switching systems we consider here. Moreover, no general conditions for the existence of a controller that makes the norm of the difference equal to 0 (and hence solves the asymptotic model matching) is given in the quoted papers, nor the problem can be easily solved by finding the controller that achieves the minimal norm and then checking if its value is 0, since the controller's construction, by linear programming techniques, is given only in the restricted cases of output or input–output switching systems and it does not appear to be generalizable.

The characterization of solvability given by Theorems 1 and 2 has a fundamental theoretical value, but in practical applications it may be difficult to check the general sufficient conditions which assure the existence of a solution and give the key ingredients for its synthesis. For this reason, in Proposition 1 and in Proposition 2, we provide sufficient solvability conditions for the SMMPSF and for the SMMPDF, respectively in the case of stability for slow switching and in the case of quadratic stability (that implies stability for arbitrary switching), that are stronger than those given in Theorems 1 and 2, but that have the advantage of being checkable by simple algorithmic procedures, as described in the comments to the above propositions. Finally, we get a stronger result by making a slightly restrictive assumption, that is akin – but definitely weaker – than invertibility, on the output difference system that compares the response of the plant with that of the model. In that situation, we characterize the solvability of the SMMPSF and of the SMMPDF (Propositions 3 and 4), basically under the same hypotheses of Theorems 1 and 2, by necessary and sufficient conditions that are checkable by simple algorithmic procedures.

The paper is organized as follows. In Section 2, we introduce notions and results of the geometric approach that are used in the sequel. In Section 3, we formally state the asymptotic matching problems we deal with, pointing out the difference between the case in which the state of the model is measurable and that in which it is not. Stability of the compensation loop is studied by considering either arbitrary switching or slow switching. In Section 4, we provide conditions for the solution of the asymptotic matching problems in the various considered cases and we illustrate the synthesis of solutions. Stronger, but practically checkable conditions for solvability are given in Section 5. An illustrative example is presented in Section 6. Section 7 contains some conclusions. The results summarized in Theorem 1 were announced in preliminary form in Conte, Perdon, and Zattoni (2014).

Notations. We denote by $\mathbb{R}$ and by $\mathbb{R}^+$ the set of real numbers and of nonnegative real numbers, respectively. Vector spaces over $\mathbb{R}$ are denoted by capital script letters (e.g. $\mathcal{X}$, $\mathcal{K}$, $\mathcal{V}$). Matrices with real entries are denoted by capital italic letters (e.g. V, L, M), vectors