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Brief paper

Partial observer normal form for nonlinear system ☆

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ABSTRACT

In this paper, we investigate the estimation problem for a class of partially observable nonlinear systems. For the proposed Partial Observer Normal Form (PONF), necessary and sufficient conditions are deduced to guarantee the existence of a change of coordinates which can transform the studied system into the proposed PONF. Examples are provided to illustrate the effectiveness of the proposed results.

1. Introduction

Observability and observer design problem for nonlinear systems have been widely studied during last four decades, and many different methods have been proposed to treat different classes of nonlinear systems, such as adaptive observer, high-gain observer, sliding mode observer and so on Besançon (2007). In this work, we are interested in the approach based on geometrical transformations to bring the original system into a simple observer normal form. The advantage of this method is that, by well choosing the desired simple observer normal form, we can reuse the existing observers proposed in the literature to estimate the state of the transformed observer normal form, and then obtain the state estimation for the original system by inverting the deduced diffeomorphism. The literature about this technique is vast. Since the pioneer works of Bestle and Zeitz (1983); Krener and Isidori (1983) for single output systems and Krener and Respondek (1985); Xia and Gao (1989) for the case of MIMO systems, many other results (see Boutat, Benali, Hammouri, & Busawon, 2009; Keller, 1987; Lynch & Bortoff, 2001; Marino & Tomei, 1996 and Phelps, 1991) were published by following the same idea. However, the solvability of the problem requires the restrictive commutative Lie bracket condition for the deduced vector fields. In order to relax this restriction, we can reconstruct a new family of vector fields which can satisfy the commutative Lie bracket condition. Inspired by this solution, various extensions are developed for output depending nonlinear observer normal form (Boutat, Zheng, Barbot, & Hammouri, 2006; Guay, 2002; Respondek, Pogromsky, & Nijmeijer, 2004; Zheng, Boutat, & Barbot, 2007, 2005, 2009) and for the extended nonlinear observer normal form (Back, Yu, & Seo, 2006; Boutat, 2015; Boutat & Busawon, 2011; Jouan, 2003; Noh, Jo, & Seo, 2004; Tami, Boutat, & Zheng, 2013). The related applications can be found as well for the synchronization of nonlinear systems Zheng and Boutat (2011), for Dengue epidemic model Tami, Boutat, Zheng, and Kratz (2014), for PM synchronous motor Tami, Boutat, and Zheng (2014), and so on.

Most of the references cited above are devoted to designing a full-order observer, under the assumption that the whole state of the studied system is observable. Few works have been dedicated to the partial observability which however makes sense in practice when only a part of states is observable or necessary for the controller design. Among the works on this issue, we can cite the work of Kang, Barbot, and Xu (2009) where the authors gave a general definition of observability covering the partial one. Reduced-order observer and LMI technique are proposed in Trinh, Fernando, and Nahavandi (2006) to estimate the part of observable states. In Robenack and Lynch (2006), the authors

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proposed a general partial nonlinear observer canonical form and used the geometrical method transforming a nonlinear dynamical system into this observer form. However, the nonlinear term in this canonical form contains as well the unobservable states. Thus, to design an observer for the proposed canonical form, one needs the Lipschitz condition to guarantee the estimation convergence. Jo and Seo (2002) provided necessary and sufficient geometrical conditions which guarantee the existence of a change of coordinates transforming a nonlinear dynamical system into a special partial observer normal form. This normal form is divided into two dynamical subsystems: the first one contains only a part of states which is of Brunovsky canonical form modulo output injection; the second one is nonlinear containing only the unobservable states and the output. This result can be seen as a direct application of the result of Krener and Isidori (1983) to the partial family of vector fields, therefore it suffers from the same restriction on the commutative Lie bracket condition.

In order to relax the restriction of the result presented in Jo and Seo (2002), this paper proposes a more general PONF for a class of partially observable nonlinear systems. This new form is a generalization of the normal form studied in Jo and Seo (2002). The proposed PONF is divided as well into two subsystems, and we relax the form proposed in Jo and Seo (2002) by involving all states in the second subsystem. To deal with this generalization, we use the notion of commutativity of Lie bracket modulo a distribution. Moreover, our results allow as well to apply additionally a diffeomorphism on the output space. Therefore, the necessary and sufficient geometrical conditions established in this paper are more general and quite different from those stated in Jo and Seo (2002) because of the introduction of commutativity of Lie bracket modulo a distribution.

The paper is organized as follows. Section 2 is devoted to the technical background and problem statement. In Section 3, some preliminary results are given and then necessary and sufficient conditions allowing the construction of PONF are stated. Section 4 generalizes the result in the previous section by applying a change of coordinates on the output. In the end, an example of Susceptible, Infected and Removed (SIR) epidemic model is presented to highlight the proposed results.

2. Notation and problem statement

We consider the following nonlinear dynamical system with single output:

$$\dot{x} = f(x) + g(x)u = f(x) + \sum_{k=1}^{m} g_k(x)\, u_k \tag{1}$$

$$y = h(x) \tag{2}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}$, and the functions $f : \mathbb{R}^n \to \mathbb{R}^n$, $g = [g_1, \ldots, g_m]$ with $g_i : \mathbb{R}^n \to \mathbb{R}^n$ for $1 \le i \le m$, $h : \mathbb{R}^n \to \mathbb{R}$ are supposed to be sufficiently smooth. In this paper, it is assumed that $f(0) = 0$ and $h(0) = 0$.

Let $\mathscr{X} \subset \mathbb{R}^n$ be a neighborhood of 0, for system (1)–(2), if the pair $(h(x), f(x))$ locally satisfies the observability rank condition on $\mathscr{X}$, i.e. rank $\{dh, dL_f h, \ldots, dL_f^n h\}(x) = n$ for $x \in \mathscr{X}$, then the following 1-forms:

$$\theta_1 = dh \quad \text{and} \quad \theta_i = dL_f^{i-1}h, \quad \text{for } 2 \le i \le n$$

are independent on $\mathscr{X}$, where $L_f^k h$ denotes the kth Lie derivative of h along f. Therefore, there exists a family of vector fields $\bar{\tau} = [\bar{\tau}_1, \ldots, \bar{\tau}_n]$ proposed in Krener and Isidori (1983), where the first vector field $\bar{\tau}_1$ is the solution of the following algebraic equations:

$$\begin{cases} \theta_i(\bar{\tau}_1) = 0 & \text{for } 1 \le i \le n-1 \\ \theta_n(\bar{\tau}_1) = 1 \end{cases} \tag{3}$$

and the other vector fields are obtained by induction as $\bar{\tau}_i = -ad_f \bar{\tau}_{i-1} = [\bar{\tau}_{i-1}, f]$ for $2 \le i \le n$, where $[,]$ denotes the Lie bracket. According to Krener and Isidori (1983), if

$$[\bar{\tau}_i, \bar{\tau}_j] = 0 \quad \text{for } 1 \le i, j \le n \tag{4}$$

$$[\bar{\tau}_i, g_k] = 0 \quad \text{for } 1 \le i \le n-1 \text{ and } 1 \le k \le m \tag{5}$$

then system (1)–(2) can be locally transformed, by means of a local diffeomorphism $\xi = \phi(x)$, into the following nonlinear observer normal form:

$$\begin{cases} \dot{\xi} = A\xi + B(y) + \displaystyle\sum_{k=1}^{m} \alpha_k(y)\, u_k \\ y = C\xi \end{cases} \tag{6}$$

where $A \in \mathbb{R}^{n\times n}$ is the Brunovsky matrix and $C = [0, \ldots, 0, 1] \in \mathbb{R}^{1\times n}$.

Obviously, for the nonlinear dynamical system (1)–(2), if rank $\{dh, dL_f h, \ldots, dL_f^n h\}(x) = r < n$ for $x \in \mathscr{X}$, which implies that only a part of states of the studied system is observable, the proposed method by Krener and Isidori (1983) could not be applied.

To treat this partially observable situation, this paper proposes the following Partial Observer Normal Form (PONF):

$$\begin{cases} \dot{\xi} = A\xi + \beta(y) + \displaystyle\sum_{k=1}^{m} \alpha_k^1(y) u_k \\ \dot{\zeta} = \eta(\xi, \zeta) + \displaystyle\sum_{k=1}^{m} \alpha_k^2(\xi, \zeta) u_k \\ y = C\xi \end{cases} \tag{7}$$

where $\xi \in \mathbb{R}^r$, $\zeta \in \mathbb{R}^{n-r}$, $y \in \mathbb{R}$, $\beta : \mathbb{R} \to \mathbb{R}^r$, $\eta : \mathbb{R}^r \times \mathbb{R}^{n-r} \to \mathbb{R}^{n-r}$, $\alpha_k^1 : \mathbb{R} \to \mathbb{R}^r$, $\alpha_k^2 : \mathbb{R}^r \times \mathbb{R}^{n-r} \to \mathbb{R}^{n-r}$, $C = (0, \ldots, 0, 1) \in \mathbb{R}^{1\times r}$ and A is the $r \times r$ Brunovsky matrix:

$$A = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \ddots & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ 0 & \cdots & \cdots & 0 & 0 \\ 0 & \cdots & \cdots & 1 & 0 \end{pmatrix} \in \mathbb{R}^{r\times r}.$$

For the proposed form (7), one can easily design a reduced-order observer to estimate the part of observable state ξ, by choosing the gain K such that $(A - KC)$ is Hurwitz. Therefore, the rest of this paper focuses only on how to deduce a diffeomorphism which transforms the nonlinear system (1)–(2) into the proposed PONF (7).

Before dealing with this problem, let us highlight the fundamental difference between the proposed PONF with respect to the canonical form proposed in Jo and Seo (2002).

Remark 1. The normal form described in (7) is a generalization of the normal form obtained in Jo and Seo (2002). Indeed, in Jo and Seo (2002) the unobservable dynamics (the second dynamics ζ) does not contain the observable state ξ. Let us consider the following modified example from Jo and Seo (2002) to highlight this difference:

$$\begin{cases} \dot{\xi}_1 = u \\ \dot{\xi}_2 = \xi_1 + y^2 \\ \dot{\zeta} = -\zeta + \mu\xi_1^2 + y \\ y = \xi_2. \end{cases} \tag{8}$$

If $\mu = 0$, then this example is the same one studied in Jo and Seo (2002) and is of the normal form proposed in Jo and Seo (2002). However, if $\mu \neq 0$, then it is of the proposed normal form (7). Moreover, the conditions stated in Jo and Seo (2002) are not fulfilled for this example when $\mu \neq 0$. This fact will be discussed in Example 1.