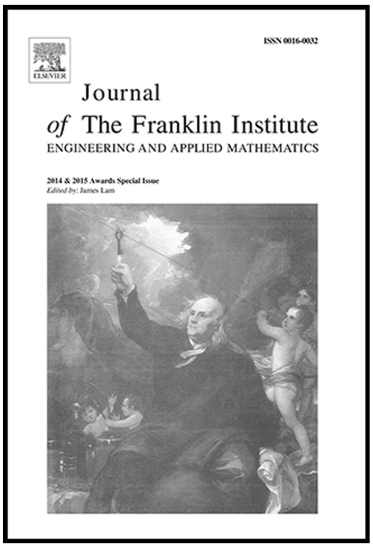

Stochastic suppression and almost surely stabilization of non-autonomous hybrid system with a new general one-sided polynomial growth condition

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Stochastic suppression and almost surely stabilization of non-autonomous hybrid system with a new general one-sided polynomial growth condition

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Abstract

This paper investigates the stochastic suppression and almost surely stabilization of non-autonomous hybrid system with a new general one-sided polynomial growth condition. Given an unstable non-autonomous hybrid system $dy(t) = f(y(t), r(t), t)dt$ with a new general one-sided polynomial growth condition, we introduce two independent Brownian noise and perturb this system into stochastic hybrid system $dx(t) = f(x(t), r(t), t)dt + q(r(t))\sqrt{\delta(t)}x(t)dB_1(t) + \sigma(r(t))\sqrt{\delta(t)}|x(t)|^\beta x(t)dB_2(t)$. It shows that the polynomial form of Brownian noise may suppress the potential explosion of hybrid system. Under a stronger condition, another linear Brownian noise will make the perturbed stochastic hybrid system is almost surely stable with general decay rate.

Keywords: Stochastic differential equations, Markovian switching, Suppression, Stabilization, Polynomial growth.

1. Introduction

It has been known that noise can be used to stabilize a given unstable system or make a system even more stable. The research work of stabilization and destabilization by noise is since 1980's [1-2]. For stability analysis of n dimension nonlinear system $dy(t) = f(y(t), t)dt, y(0) = y_0$ under f satisfies the local Lipschitz condition, Mao investigated the stabilization by linear form of Brownian noise when f satisfies linear growth condition [3]. Then Appleby et al. examined the stabilization of noise for functional system $\dot{x}(t) = f(x_t, t)$, where $x_t = x_t(\theta) := \{x(t+\theta) : -\bar{\tau} \le \theta \le 0\}$ and $f : C([-\bar{\tau}, 0]; R^n) \times R_+ \to R^n$ satisfies the one-sided linear growth condition [4-5]. Details on stabilization of stochastic system by linear form of Brownian noise can be found in [6]. The results of stabilization by linear form Brownian noise are extended to hybrid system in continuous time state observation feedback control [7-8], delay feedback control [9], discrete time observation feedback control [10-12]. For noise suppression effect, Mao et. al [13] showed environmental noise can suppress the explosion of population dynamics system in finite time. Deng et. al revealed an important feature that noise can suppress or express exponential growth while f satisfies the one-sided linear growth condition [14]. The results of suppress effect of noise are then extended to hybrid system in papers [15-16].

To cope with the restriction of linear growth condition or one-sided linear growth condition for stochastic system, Luo, Mao, and Shen proposed general conditions using probability measures functions method [17]; Hu, Mao, and Zhang [18] proposed polynomial restriction term and M matrices for hybrid stochastic system; Wu and Hu [19] proposed one-sided polynomial growth condition that is $y^T f(y,t) \le \rho|y|^{\alpha+2} + \kappa|y|^2, \forall (y,t) \in R^n \times R_+$ where ρ, κ, α are nonnegative constants, and investigated the suppression and stabilization of noise. They used two independent Brownian noise feedback terms and perturbed it as $dx(t) = f(x(t), t)dt + qx(t)dB_1(t) + \sigma|x(t)|^\beta x(t)dB_2(t)$, it was shown that the nonlinear noise may suppress

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