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Brief paper

Barrier Lyapunov Functions-based adaptive control for a class of nonlinear pure-feedback systems with full state constraints☆

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ABSTRACT

In this study, an adaptive control technique is developed for a class of uncertain nonlinear parametric systems. The considered systems can be viewed as a class of nonlinear pure-feedback systems and the full state constraints are strictly required in the systems. One remarkable advantage is that only less adjustable parameters are used in the design. This advantage is first to take into account the pure-feedback systems with the full state constraints. The characteristics of the considered systems will lead to a difficult task for designing a stable controller. To this end, the mean value theorem is employed to transform the pure-feedback systems to a strict-feedback structure but non-affine terms still exist. For the transformed systems, a novel recursive design procedure is constructed to remove the difficulties for avoiding non-affine terms and guarantee that the full state constraints are not violated by introducing Barrier Lyapunov Function (BLF) with the error variables. Moreover, it is proved that all the signals in the closed-loop system are global uniformly bounded and the tracking error is remained in a bounded compact set. Two simulation studies are worked out to show the effectiveness of the proposed approach.

1. Introduction

In real world, the practical plants are inevitable to contain the uncertainties. Driven by practical requirements and theoretical challenges, the controller design of uncertain systems has become an important research domain (Li, Gao, Shi, & Zhao, 2014; Li, Pan, & Zhou, 2015; Li, Yin, Pan, & Lam, 2015). Adaptive control algorithms on nonlinear systems with uncertain parameters have received much attention (Krstic, Kanellakopoulos, & Kokotovic, 1995; Krstic & Kokotovic, 1996; Wang & Lin, 2010, 2012; Yang, Ge, & Lee, 2009; Zhang & Lin, 2015; Zhang & Xu, 2015). Subsequently, some significant works on adaptive NN control for nonlinear systems with unknown functions were revealed in Chen, Hua, and Ge (2014), Ge and Wang (2004), Liu and Tong (2015) and Tong, Sui, and Li (2015); Tong, Zhang, and Li (in press). Other notable adaptive control methods were proposed in He, Ge, How, Choo, and Hong (2011); He, Zhang, and Ge (2014a,b) and Li, Xiao, Yang, and Zhao (2015) for some real systems with uncertain parameters. Adaptive control for a more general class of pure-feedback systems with the uncertainties was studied in Ge and Wang (2002), Ge, Yang, and Lee (2008), Krstic et al. (1995) and Nam and Araposthathis (1988). However, the effect of the constraints is omitted in the above-mentioned results.

It is a fact that many real systems suffer from the effect of the constraints, such as the temperature of chemical reactor and physical stoppages. Two control approaches were proposed in Zhou, Duan, and Lin (2011); Zhou, Li, and Lin (2013) for linear discrete-time systems with input constraints subject to actuator saturation. In Chen, Ge, and Ren (2011), an adaptive tracking control was constructed for uncertain multi-input and multi-output nonlinear systems subject to non-symmetric input constraints. The effect of input constraints is overcome by introducing an auxiliary design system. But these approaches cannot solve the control problem of the state constraints.

Recently, the utilization of BLF for solving the control problem of nonlinear systems with output and state constraints has been an active area. A survey paper on constraint-handling ways of model predictive control (Mayne, Rawlings, Rao, & Scokaert, 2000) was well established. It has also employed BLFs to design the adaptive controller for nonlinear strict-feedback systems with the time-unvarying (Tee, Ge, & Tay, 2009a) and time-varying (Tee, Ren, & Ge, 2011) output constraints. In Ren, Ge, Tee, and Lee (2010),

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an adaptive neural control was presented for a class of nonlinear systems with unknown functions, immeasurable states and the output constraint. A BLF is effectively used in the output-feedback control. Moreover, the control problem of some practical systems-based BLF has been coped with, such as a flexible crane system (He et al., 2014a,b). Besides the output constraint mentioned in the above works, the state constraints have also been tackled by using BLF. Adaptive control techniques-based on BFLs have been addressed for nonlinear strict-feedback with partial state constraints (Tee & Ge, 2011) and the full state constraints (Liu, Li, & Tong, 2014; Tang, 2014; Tee & Ge, 2012; Tee, Ge, & Tay, 2009b). These techniques on state constraints can be only to control a class simple of nonlinear strict-feedback systems. To guarantee the stability of more general class of systems, an adaptive neural control approach was studied in Kim and Yoo (2014) for nonlinear pure-feedback systems with full state constraints. However, the adaptive design of this approach is obtained based on the neural weight vector. When the neural network nodes are increased, the number of adjustable parameters will be enormous. Then, the on-line learning time becomes very large.

In this paper, an adaptive control technique is studied for a class of nonlinear parametric systems. The main contributions of the proposed approach are that:

(1) Different from the results in Liu et al. (2014), Tee and Ge (2011) and Tee et al. (2009b) which focused on the strict-feedback systems with state constraints, this paper frames a generalization of the results for a more general class of nonlinear pure-feedback systems with the full state constraints. At the same time, in contrast to the approach in Kim and Yoo (2014), less adjustable parameters are used in the design. It is first to design an adaptive control technique with less adjustable parameters to treat the full state constraints of nonlinear pure-feedback systems.
(2) In order to control this class of systems, the pure-feedback systems are transformed into a strict-feedback structure with non-affine terms by using the mean value theorem. For the transformed systems, a modified backstepping design based on BLFs is proposed to prevent the violation of the full state constraints and the adaptation laws for the estimations of norms on uncertain parameters are constructed.

Finally, it is proved that all the signals in the closed-loop system are global uniformly bounded and the tracking error is remained in a compact set. The effectiveness of the approach can be shown by working out two simulation examples.

2. System description and preliminaries

Consider the nonlinear pure-feedback systems (Ge & Wang, 2002)

$$\begin{cases}\dot{x}_i = f_i\left(x_1,\ldots,x_{i+1}\right), \quad i=1,\ldots,n-1\\ \dot{x}_n = f_n\left(x,u\right)\\ y = x_1\end{cases} \tag{1}$$

where $x = [x_1, \ldots, x_n]^T \in R^n$, $y \in R$ and $u \in R$ are the states, the output, the input of the systems, respectively; all the states are constrained in the compact sets, i.e., x_i is required to remain in the set $|x_i| < k_{c_i}$ with k_{c_i} being a positive constant; let $\bar{x}_i = [x_1, \ldots, x_i]^T \in R^i$; $f_i\left(\bar{x}_{i+1}\right)$, $i = 1, \ldots, n-1$ and $f_n(x,u)$ are nonlinear smooth functions and continuously differentiable with respect to the argument x_{i+1} and u, respectively. Let $\mu_i\left(\bar{x}_{i+1}\right) = \partial f_i\left(\bar{x}_{i+1}\right)/\partial x_{i+1}$ and $\mu_n(x,u) = \partial f_n(x,u)/\partial u$.

By using the mean value theorem (Ge & Wang, 2002), there must exist x_{i+1}^0 and u^0 such that

$$f_i\left(\bar{x}_{i+1}\right) = f_i\left(\bar{x}_i, 0\right) + \mu_i\left(\bar{x}_i, x_{i+1}^0\right)x_{i+1}, \quad i=1,\ldots,n-1 \tag{2}$$

$$f_n(x,u) = f_n(x,0) + \mu_n\left(x,u^0\right)u \tag{3}$$

where x_{i+1}^0 is some point between zero and x_{i+1}, and u^0 is some point between zero and u.

In this paper, the nonlinear smooth functions $f_i\left(\bar{x}_{i+1}\right)$, $i = 1, \ldots, n-1$ and $f_n(x,u)$ are uncertain, and they satisfy the following condition

$$f_i\left(\bar{x}_{i+1}\right) = \theta_i^T\gamma_i\left(\bar{x}_{i+1}\right), \quad i=1,\ldots,n-1 \tag{4}$$

$$f_n(x,u) = \theta_n^T\gamma_n(x,u) \tag{5}$$

where $\theta_i \in R^m$ is an uncertain constant vector; $\gamma_i\left(\bar{x}_{i+1}\right)$, $i = 1, \ldots, n-1$ and $\gamma_n(x,u)$ are known continuous function vectors.

The design objective of this study is to construct an adaptive state feedback controller u such that the tracking error $z_1 = y - y_d(t)$ converges to a bounded compact set, all the signals in the closed-loop system are global uniformly bounded and the full state constraints are not violated where $y_d(t) \in R$ is the reference signal to be known and bounded.

To achieve the control objective, it is necessary to make the following assumptions.

Assumption 1 (*Ge & Wang, 2002*). The functions $\mu_i(\cdot)$, $i = 1, \ldots, n$ are bounded, i.e., there exist the constants $\overline{\mu}_i \geq \underline{\mu}_i > 0$ such that $\underline{\mu}_i \leq |\mu_i(\cdot)| \leq \overline{\mu}_i$. Without losing generality, this paper assumes that $\underline{\mu}_i \leq \mu_i(\cdot) \leq \overline{\mu}_i$.

Assumption 2 (*Tee & Ge, 2011*). It is assumed that $y_d(t)$ and its ith order derivatives $y_d^{(i)}(t)$, $i = 1, \ldots, n$ satisfy $|y_d(t)| \leq A_0 < k_{c_1}$ and $\left|y_d^{(i)}(t)\right| \leq A_i$ where $A_0, A_1, \ldots, A_n$ are positive constants.

Remark 1. In Tee and Ge (2011, 2012) and Tee et al. (2009a, 2011), adaptive output and state constraint control problems were addressed for uncertain nonlinear strict-feedback systems, i.e., $f_i(.)$ is a function of $\bar{x}_i$. From (1), it can be seen that $f_i(.)$ is not only a function of $\bar{x}_i$, but also a function of x_{i+1} with $u = x_{n+1}$. Thus, the system structure is more general than the previous results.

3. The controller design and stability analysis

Define the tracking error $z_1 = y - y_d(t)$ and the variables $z_i = x_i - \alpha_{i-1}$, $i = 2, \ldots, n$ where α_{i-1} is a virtual controller to be designed in *Step i*. The detailed backstepping design is given as follows.

Step 1: The time derivative of $z_1 = x_1 - y_d(t)$ is

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_d(t) = f_1(x_1,x_2) - \dot{y}_d(t). \tag{6}$$

Using (2) and (4), (6) becomes

$$\dot{z}_1 = \theta_1^T\gamma_1(x_1) + \mu_1\left(x_1,x_2^0\right)x_2 - \dot{y}_d(t). \tag{7}$$

Introduce the variable $z_2 = x_2 - \alpha_1$ and we have

$$\dot{z}_1 = \theta_1^T\gamma_1(x_1) + \mu_1\left(x_1,x_2^0\right)(z_2+\alpha_1) - \dot{y}_d(t). \tag{8}$$

The virtual controller α_1 is designed as

$$\alpha_1 = -\rho_1 z_1 - \hat{\vartheta}_1 K_{z_1}\Upsilon_1(x_1)/2\delta_1^2 - K_{z_1}\Phi_1/2 \tag{9}$$

where ρ_1 and δ_1 are the positive design parameters, $\Upsilon_1(x_1) = \|\gamma_1(x_1)\|^2$, $\Phi_1 = (\dot{y}_d(t))^2$, $K_{z_1} = z_1/\left(k_{b_1}^2 - z_1^2\right)$, $\hat{\vartheta}_1 > 0$ is the estimation of $\vartheta_1 = \|\theta_1\|^2$, $k_{b_1} = k_{c_1} - A_0$ is a positive constant. In the following, $K_{z_i} = z_i/\left(k_{b_i}^2 - z_i^2\right)$, $i \geq 2$ will be used and define a compact set $\Omega_{z_i} := \left\{|z_i| < k_{b_i}\right\}$, $i = 1, \ldots, n$ where k_{b_i} is specified later on.