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Brief paper

Distributed finite-time tracking of multiple non-identical second-order nonlinear systems with settling time estimation[☆]

Yu Zhao[a], Zhisheng Duan[b], Guanghui Wen[c], Guanrong Chen[d]

[a] Department of Traffic and Control Engineering, School of Automation, Northwestern Polytechnical University, Xi'an Shaanxi, 710129, China

[b] State Key Laboratory for Turbulence and Complex Systems, Department of Mechanics and Engineering Science, College of Engineering, Peking University, Beijing 100871, China

[c] Department of Mathematics, Southeast University, Nanjing 210096, China

[d] Department of Electronic Engineering, City University of Hong Kong, Hong Kong Special Administrative Region

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ABSTRACT

This paper investigates the distributed finite-time consensus tracking problem for a group of autonomous agents modeled by multiple non-identical second-order nonlinear systems. First, a class of distributed finite-time protocols are proposed based on the relative position and relative velocity measurements. By providing a topology-dependent Lyapunov function, it is shown that distributed consensus tracking can be achieved in finite time under the condition that the nonlinear errors between the leader and the followers are bounded. Then, a new class of observer-based algorithms are designed to solve the finite-time consensus tracking problem without using relative velocity measurements. The main contribution of this paper is that, by computing the value of the Lyapunov function at the initial point, the finite settling time can be theoretically estimated for second-order multi-agent systems with the proposed control protocols. Finally, the effectiveness of the analytical results is illustrated by an application in low-Earth-orbit spacecraft formation flying.

1. Introduction

During the past two decades, distributed cooperative control of autonomous agents has emerged as a new research direction and received increasing interest in different fields with the advent of wireless communication networks and powerful embedded systems. Research on this topic aims to understand how various group behaviors emerge as a result of local interactions among individuals. Distributed cooperative control has applications in a wide range of areas, such as attitude synchronization, state consensus, formation flying, and cooperative surveillance (Chen, Liu, & Lu, 2007; Hong, Chen, & Bushnell, 2008; Li, Duan, Chen, & Huang, 2010; Ren, Beard, & Atkins, 2007).

In a distributed cooperative control system, a group of autonomous agents, by coordinating with each other via communication or sensing networks, can perform certain challenging tasks which cannot be well accomplished by a single agent. As one of the important and fundamental research issues for multi-agent systems, consensus problem has been extensively studied over the past few years. The objective is to develop distributed control policies using only local relative information to ensure that the states of the agents reach an agreement on certain quantities of interest. A pioneering work on consensus was attributed to Olfati-Saber and Murray (2004), where a general framework of the consensus problem for networks of integrators was proposed. Since then, a variety of consensus algorithms have been proposed to solve the consensus problem under different scenarios; see Cao, Morse, and Anderson (2008), Hong et al. (2008), Li et al. (2010), Ren and Beard (2005), Xiao, Wang, Chen, and Gao (2009), Yu, Chen, and Cao (2007) and references therein. According to the number of leaders in the network, existing consensus algorithms can be roughly categorized into two classes, namely, leaderless consensus and leader-following consensus. The latter is also called the distributed tracking problem, where the objective is to drive the states of

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E-mail addresses: yuzhao5977@gmail.com (Y. Zhao), duanzs@pku.edu.cn (Z. Duan), wenguanghui@gmail.com (G. Wen), eegchen@cityu.edu.hk (G. Chen).

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the followers to track those of the leader. Seminal works on the consensus tracking problem with integrator-type dynamics include Cao and Ren (2012) and Hong et al. (2008), where some effective algorithms were proposed for followers to track a dynamic leader. Then, the works in Li et al. (2010); Li, Liu, Ren, and Xie (2013) extended the dynamics of agents to general linear systems. Moreover, various distributed tracking problems for multiple nonlinear systems were studied in Hu (2011, 2012), Meng, Lin, and Ren (2013) Song, Cao, and Yu (2010), Su, Chen, Wang, and Lin (2011) and Yu, Chen, Cao, and Kruths (2010).

Convergence rate, as a significant performance index for evaluating the effectiveness of the designed distributed algorithms, is a focal research topic in the study of consensus problems. Numerous researchers endeavored to improve the convergence rate by enlarging the coupling strength, optimizing the system gain, or designing a better communication topology (Li, Duan, & Chen, 2011; Zhao, Duan, & Wen, 2015). However, the above-mentioned methods may only guarantee asymptotic consensus. In practical applications, it is often desirable to achieve consensus in finite time. Finite-time consensus problem was first studied in Cortés (2006), where the agents have first-order dynamics. Then, by using tools from homogeneity theory, finite-time consensus problems with nonlinear dynamics were investigated in Wang and Hong (2008). Recently, finite-time leader-following consensus problems for second-order multi-agent systems have been investigated in Li, Du, and Lin (2011) and Zhao, Duan, Wen, and Zhang (2013). It is worth noting that most of the above-mentioned works (Cortés, 2006; Li, Du et al., 2011; Wang & Hong, 2008; Zhao et al., 2013) are derived based on a common assumption that all agents have identical dynamics. Generally, this assumption is too strict in practice. Generally, the above-mentioned results cannot be directly used to solve the distributed tracking problem for multiple non-identical second-order nonlinear systems.

Motivated by the above observations, this paper investigates the distributed finite-time tracking problem for a group of multi-agent systems with non-identical second-order nonlinear dynamics. Two challenging cases are considered: when both the relative position and relative velocity measurements are available, and when only the relative position measurements are available. The main results of this paper extend the existing works on distributed consensus tracking of multiple second-order nonlinear systems in three aspects. First, a class of distributed finite-time tracking algorithms are proposed that can be used for the case where the follower dynamics are heterogeneous and the trajectory of the time-varying leader is arbitrary as long as the nonlinear errors among the agents are bounded. This relaxes the assumptions that the follower dynamics are identical and the dynamics of the leader are the same as those of the followers with a zero control input, as made in Hu (2011, 2012), Li, Du et al. (2011), Song et al. (2010), Su et al. (2011) and Zhang and Lewis (2012). Then, a distributed finite-time tracking protocol is designed by using only relative position measurements, where the velocities of the followers are assumed to be unavailable. This extends the existing works on the case of the state feedback as studied in Hu (2011, 2012), Song et al. (2010), Su et al. (2011), Yu et al. (2010) and Zhang and Lewis (2012), and provides an efficient strategy for applications with low-cost configurations, where velocity measurement sensors are not needed. Third, the estimation of the finite time can be obtained by computing the value of a carefully designed Lyapunov function at the initial point. This extends the existing works on asymptotic consensus as in Meng et al. (2013) and finite-time consensus as discussed in Zhao, Duan, Wen, and Chen (2015); Zhao et al. (2013). In addition, it is theoretically significant and practically important to provide an estimation on the settling time of consensus tracking, which is another achievement of the present paper.

The remainder of this paper is organized as follows. The preliminaries and the problem formulation are given in Section 2. Main theoretical results are established in Sections 3 and 4. In Section 5, some numerical simulations are reported to illustrate the theoretical results. Concluding remarks are finally given in Section 6.

2. Preliminaries and model description

Notations: Let $\mathbb{R}^{n\times n}$ be the set of $n \times n$ real matrices, $\mathbb{R}^+$ the set of positive real numbers and I_p the p-dimensional identity matrix. $P > 0$ ($P < 0$) means that the matrix P is positive (negative) definite. $\mathbf{1}$ represents the vector with all entries being one. Let $|t|$ be absolute value of a scalar t. Given a vector $\xi = [\xi_1, \xi_2, \ldots, \xi_p]^T$, define $\|\xi\|_q = (\sum_{i=1}^{p} |\xi_i|^q)^{\frac{1}{q}}$ with $q > 0$, $\text{sgn}(\xi) = [\text{sgn}(\xi_1), \text{sgn}(\xi_2), \ldots, \text{sgn}(\xi_p)]^T$ and $\text{sig}(\xi)^{1/2} = [|\xi_1|^{1/2}\text{sgn}(\xi_1), |\xi_2|^{1/2}\text{sgn}(\xi_2), \ldots, |\xi_p|^{1/2}\text{sgn}(\xi_p)]^T$, where $\text{sgn}(\cdot)$ is the signum function. Let $\text{diag}(\xi)$ represent a diagonal matrix with diagonal elements $\xi_1, \xi_2, \ldots, \xi_p$. $\|T\|_\infty$ denotes the infinite norm of matrix T. The Kronecker product of matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$ is denoted by $A \otimes B$.

Consider a group of N agents. Denote by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ a communication topology among N agents (nodes), where $\mathcal{V} = \{1, 2, \ldots, N\}$ and $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ represent the set of nodes and the set of edges, respectively. A directed edge $(i, j) \in \mathcal{E}$ in graph $\mathcal{G}$ means that agent i can obtain information from agent j, but not conversely. The neighborhood of agent i is denoted as $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$. Furthermore, a graph is undirected if $(i, j) \in \mathcal{E}$ implies $(j, i) \in \mathcal{E}$. An undirected edge $(i, j) \in \mathcal{E}$ in graph $\mathcal{G}$ means that agent i and j can access information from each other. In this paper, it is assumed that $\mathcal{G}$ is a simple graph, i.e., $(i, i) \notin \mathcal{V}$, which indicates that each agent cannot use its own measurements for feedback. A path from agent i_1 to agent i_s is a sequence of ordered edges in the form of $(i_k, i_{k+1}) \in \mathcal{E}$, $k = 1, 2, \ldots, s-1$. A graph $\mathcal{G}$ is said to be connected if there exists a path among each pair of distinct nodes. The adjacency matrix of a graph $\mathcal{G}$ is denoted by $A = (a_{ij}) \in \mathbb{R}^{N\times N}$, where $a_{ij} = 1$ if $(i, j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix of the graph $\mathcal{G}$ associated with the adjacency matrix A is designed as $\mathcal{L} = (l_{ij})$, where $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$, $i, j = 1, 2, \ldots, N$.

In this paper, agents labeled as $1, 2, \ldots, N-1$, are followers while agent N is the leader. Suppose the leader has no neighbor. Thus, $\mathcal{L}$ can be rewritten as

$$\mathcal{L} = \begin{bmatrix} L_1 & b \\ 0 & 0 \end{bmatrix}, \quad (1)$$

where $L_1 = L_f - \text{diag}(b) \in \mathbb{R}^{(N-1)\times(N-1)}$ is a square matrix, in which L_f represents the Laplacian matrix among the followers and $b \in \mathbb{R}^{(N-1)\times 1}$ is a column vector. Throughout, it is assumed that the network communication topology among the followers is undirected. For each follower, there exists at least one directed path from the follower to the leader.

Before moving on, the following lemmas are provided.

Lemma 1 (*Chen et al., 2007, Li et al., 2013, Song et al., 2010*). *Under above-mentioned communication topology, $b \in \mathbb{R}^{(N-1)\times 1}$ has at least one negative entry and L_1 is symmetric and positive definite.*

Lemma 2 (*David, 1987*). *For real numbers $a > 0, b > 0, c > 0, p > 1, q > 1$, with $\frac{1}{p} + \frac{1}{q} = 1$, the following inequality is satisfied:*

$$ab \le c^p \frac{a^p}{p} + c^{-q} \frac{b^q}{q}.$$

Lemma 3 (*David, 1987*). *For real numbers $a \ge 0, b \ge 0$ and $0 < p < q$, the following inequality is satisfied:*

$$(a^q + b^q)^{\frac{1}{q}} \le (a^p + b^p)^{\frac{1}{p}}.$$