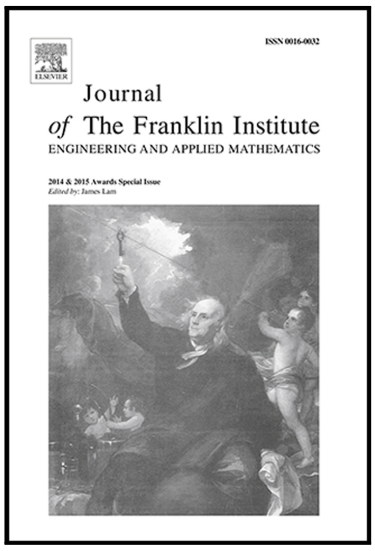

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Accelerated Smith Iterative Algorithms for Coupled Lyapunov Matrix Equations

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Abstract

In this paper, a kind of explicit iterative algorithm is studied by using Smith methods for solving coupled Lyapunov matrix equations arising in continuous-time Markovian jump linear systems. First, by introducing the latest estimation, a current-estimation-based Smith iterative algorithm is presented to solve the considered matrix equations. The monotonicity and boundedness of the sequence generated by this algorithm with zero initial conditions are shown by the mathematical induction, and a convergence result is accordingly obtained based on these two properties. To further improve the convergence performance, an accelerated Smith iterative algorithm is proposed. Convergence analysis indicates the iterative solutions generated by the developed algorithms always converge to the unique solutions of the coupled Lyapunov matrix equations for any initial conditions. A numerical example is provided to show the effectiveness of the presented iterative algorithms.

Keywords: Smith methods, iterative solutions, coupled Lyapunov matrix equations.

1 Introduction

Over the last decades, various types of matrix equations have received much attention due to its extensive applications [1–4]. In particular, the coupled Lyapunov matrix equations play an vital role in stability analysis and stabilizing controller design for Markovian jump linear systems. It has been shown in [2] and [5] that the stochastic stability of the Markovian jump linear system can be characterized by the existence of unique positive definite solutions of the coupled Lyapunov matrix equations. In [6], the relation between the H_2 norm and the solutions of coupled Lyapunov matrix equations was established.

A lot of attention has already been focused on the solving methods of the matrix equations associated with linear system theory [7, 8]. As a matter of fact, the exact solutions of matrix equations can be obtained from matrix inversion by using the Kronecker product. With this idea, a direct method was established in [9] for solving the coupled matrix equations. However, the algorithm in [9] requires excessive computer storage and is of limited use for large scale systems. In order to solve this problem, iterative approaches have been widely investigated in many literatures for matrix equations [10–13]. In [14], an iterative algorithm has been proposed to solve the coupled Lyapunov matrix equations related to Markovian jump linear systems for the first time. A main feature of the algorithm presented in [14] is that some standard Lyapunov equations need to be solved at each iterative step. Due to this, the algorithm in [14] is in an implicit iterative form. Recently, a modified version of the preceding implicit iterative algorithm was developed in [15] by introducing the latest estimation. More recently, a weighted implicit iterative algorithm was developed in [16] for solving the coupled Lyapunov matrix equations. The convergence rate of the weighted iterative algorithm in [16] can be significantly improved if the weighting parameters are appropriately chosen. However, the implicit iterative algorithms in [15, 16] also require the solutions of the standard Lyapunov equations at each iterative step. It is well known that for high dimension matrices the solution of the standard Lyapunov equation is still too costly. For such reason, some explicit iterative algorithms, such as the gradient-based iterative algorithms, have attracted much attentions for solving the coupled Lyapunov matrix equations. In [17] and [18], some gradient-based algorithms have been investigated to solve the coupled matrix equations, including the coupled Lyapunov matrix equations as a special case. In [19], some reduced-rank gradient-based iterative algorithms for generalized coupled Sylvester matrix equations were constructed. It should be noted that the proposed algorithms in [19] also can be used to solve the coupled Lyapunov matrix equations. Recently, some novel explicit iterative algorithms