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Brief paper

On the passivity based control of irreversible processes: A port-Hamiltonian approach ☆

Héctor Ramírez [a], Yann Le Gorrec [a], Bernhard Maschke [b], Françoise Couenne [b]

[a] FEMTO-ST, UMR CNRS 6174, AS2M Department, Université de Bourgogne Franche-Comté, Université de Franche-Comté, ENSMM, 24 rue Savary, F-25000 Besanon, France

[b] LAGEP, UMR CNRS 5007, Université de Lyon, Université Lyon 1, Faculté Sciences et Technologie, 43 Boulevard du 11 Novembre 1918, F-69622 Villeurbanne, France

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ABSTRACT

Irreversible port-Hamiltonian systems (IPHS) have recently been proposed for the modelling of irreversible thermodynamic systems. On the other hand, a classical result on the use of the second law of thermodynamics for the stabilization of irreversible processes is the celebrated thermodynamic availability function. These frameworks are combined to propose a class of Passivity Based Controller (PBC) for irreversible processes. An alternative formulation of the availability function in terms of internal energy is proposed. Using IPHS a matching-condition, which is interpreted in terms of energy-shaping, is derived and a specific solution that permits to assign a desired closed-loop structure and entropy rate is proposed. The approach can be compared with Interconnection and Damping Assignment-PBC, this method however leads in general to thermodynamically non-coherent closed-loop systems. In this paper a system theoretic approach is employed to derive a constructive method for the control design. The closed-loop system is in IPHS form, hence it can be identified with a thermodynamic system and the control parameters related with thermodynamic variables, such as the reaction rates in the case of chemical reactions. A generic non-linear non-isothermal continuous stirred tank reactor is used to illustrate the approach.

1. Introduction

The different suggestions for the modelling of irreversible thermodynamic processes as (dissipative) port-Hamiltonian systems (PHS) (Duindam, Macchelli, Stramigioli, & Bruyninckx, 2009; Maschke & van der Schaft, 1992; van der Schaft & Maschke, 1995) have led to a class of system called *quasi-PHS* (Dörfler, Johnsen, & Allgöwer, 2009; Eberard, Maschke, & van der Schaft, 2007; Hangos, Bokor, & Szederkényi, 2001; Hoang, Couenne, Jallut, & Le Gorrec, 2011; Otero-Muras, Szederkényi, Alonso, & Hangos, 2008; Ramirez, Sbarbaro, & Ortega, 2009). These systems retain much of the dissipative port Hamiltonian structure, but differ by their structure (interconnection and dissipation) matrices and input vector fields which depend explicitly on the gradient of the Hamiltonian. This framework has recently been combined with the framework of the thermodynamic availability function (Alonso & Ydstie, 1996, 2001; Ydstie & Alonso, 1997) to derive Lyapunov conditions for the stabilization of irreversible thermodynamic systems (Hoang et al., 2011; Hoang, Couenne, Jallut, & Le Gorrec, 2012; Ydstie, 2002). From a control design perspective this implies that when looking for closed-loop potentials, for instance when passivity based control (PBC) techniques are applied (Ortega, van der Schaft, Mareels, & Maschke, 2001; Ortega, van der Schaft, Maschke, & Escobar, 2002), the integrability conditions lead to partial differential equations which are nonlinear instead of linear. Furthermore, it is well known that for this case a physically consistent parametrization of the control problem is far from obvious (Kotyczka, 2013). This implies closed-loop systems without physical interpretation or very complex matching equations to solve during the design.

In this paper we shall consider the control of a class of such extensions of PHS, named Irreversible Port-Hamiltonian Systems (IPHS) (Ramirez, Maschke, & Sbarbaro, 2013a,b). These systems

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E-mail addresses: hector.ramirez@femto-st.fr (H. Ramírez), legorrec@femto-st.fr (Y. Le Gorrec), maschke@lagep.univ-lyon1.fr (B. Maschke), couenne@lagep.univ-lyon1.fr (F. Couenne).

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embed by construction simultaneously the first (conservation of energy) and the second principle (irreversible creation of entropy). An incremental energy function, defined as an energy based availability function, is used as desired closed-loop Hamiltonian following Ramirez, Gorrec, Maschke, and Couenne (2013); Ramirez, Le Gorrec, and Maschke (2014). A Lyapunov condition is then derived and interpreted in terms of *energy-shaping* passivity based control (PBC) (Ortega et al., 2001, 2002). The Lyapunov condition is then further developed and a specific non-linear solution, which permits to assign a desired closed-loop interconnection structure and entropy dissipation rate, is proposed.

The proposed design procedure consists in finding appropriate structure matrices and desired thermodynamic control functions to solve *algebraically* (Acosta, Ortega, Astolfi, & Sarras, 2008; Nunna, Sassano, & Astolfi, 2015) the associated matching equations. The IPHS formulation allows to systematically parametrize the problem to derive the conditions for a globally stabilizing controller which preserves the IPHS structure in closed-loop. Since the structure of the closed-loop system is IPHS, it can be interpreted as a thermodynamic system and the parameters of the controller related with thermodynamic variables, such as the reaction rates in the case of chemical reactions.

The paper is organized as follows: Section 2 recalls the definition and physical interpretation of IPHS. In Section 3 the framework of the thermodynamic availability function is presented and a general Lyapunov condition is derived. Section 4 presents the main results of this paper. In Section 5 the approach is applied to the example of a generic non-linear non-isothermal CSTR model. Finally Section 6 gives some closing remarks and comments on future work.

2. Irreversible Port-Hamiltonian systems

Irreversible Port Hamiltonian Systems (IPHS) have been defined in Ramirez, Maschke, et al. (2013a) as an extension of Port Hamiltonian systems for the purpose of representing not only the energy balance but also the entropy balance, essential in thermodynamic systems.

Definition 1 (*Ramirez, Maschke, et al., 2013a*). An input affine IPHS is defined by the dynamic equation and output relation

$$\begin{aligned} \dot{x} &= R\left(x, \tfrac{\partial U}{\partial x}, \tfrac{\partial S}{\partial x}\right) J \frac{\partial U}{\partial x}(x) + g\left(x, \tfrac{\partial U}{\partial x}\right) u, \\ y &= g^\top\left(x, \tfrac{\partial U}{\partial x}\right) \frac{\partial U}{\partial x}(x) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, the smooth functions $U(x) : \mathbb{R}^n \to \mathbb{R}$ and $S(x) : \mathbb{R}^n \to \mathbb{R}$ represent, respectively, the internal energy (the Hamiltonian) and the entropy functions, $J \in \mathbb{R}^{n\times n}$ is a constant skew-symmetric structure (interconnection) matrix of the Poisson bracket (Maschke, van der Schaft, & Breedveld, 1992) acting on any two smooth functions Z and G as:

$$\{Z, G\}_J = \frac{\partial Z}{\partial x}^\top(x) J \frac{\partial G}{\partial x}(x). \tag{2}$$

The real function $R = R\left(x, \frac{\partial U}{\partial x}, \frac{\partial S}{\partial x}\right)$ is composed by the product of a positive definite function γ and the Poisson bracket between the entropy and the energy functions:

$$R\left(x, \tfrac{\partial U}{\partial x}, \tfrac{\partial S}{\partial x}\right) = \gamma\left(x, \tfrac{\partial U}{\partial x}\right)\{S, U\}_J,$$

with $\gamma\left(x, \frac{\partial U}{\partial x}\right) : \mathbb{R}^n \to \mathbb{R}$, $\gamma \geq 0$, a non-linear positive function. The input map is defined by $g\left(x, \frac{\partial U}{\partial x}\right) \in \mathbb{R}^{n\times m}$ with the input $u(t) \in \mathbb{R}^m$ a time dependent function.

The drift dynamic in (1) is defined by a non-linear relation between the time derivative $\dot{x}$ of the state variables and $\frac{\partial U}{\partial x}$, characterized by the modulating function $R\left(x, \frac{\partial U}{\partial x}, \frac{\partial S}{\partial x}\right)$, which explicitly depends on the differential of the energy $\frac{\partial U}{\partial x}$. The balance equations of the total energy and entropy functions of IPHS express the first and second principles of irreversible thermodynamics: the conservation of energy and the irreversible creation of entropy due to irreversible phenomena. By skew-symmetry of J, the balance equation of the internal energy,

$$\frac{dU}{dt} = y^\top u, \tag{3}$$

expresses that the system (1) is a lossless dissipative systems with (energy) supply rate $y^\top u$ (Willems, 1972). The balance equation of the entropy function is given by

$$\begin{aligned} \frac{dS}{dt} &= R\left(x, \tfrac{\partial U}{\partial x}, \tfrac{\partial S}{\partial x}\right) \frac{\partial S}{\partial x}^\top J(x) \frac{\partial U}{\partial x} + \frac{\partial S}{\partial x}^\top g\left(x, \tfrac{\partial U}{\partial x}\right) u \\ &= \gamma\left(x, \tfrac{\partial U}{\partial x}\right)\{S, U\}_J^2 + \left(g^\top\left(x, \tfrac{\partial U}{\partial x}\right) \frac{\partial S}{\partial x}\right)^\top u. \end{aligned} \tag{4}$$

By Definition 1 the first term is positive: $\gamma\left(x, \frac{\partial U}{\partial x}\right)\{S, U\}_J^2 = \sigma\left(x, \frac{\partial U}{\partial x}\right) \geq 0$. For irreversible thermodynamic systems, this term represents the *internal entropy production* and its positivity expresses the second principle of thermodynamics. The second term in (4) corresponds to the definition of an entropy supply rate. For further details on IPHS and its thermodynamic interpretation we refer the reader to Ramirez, Maschke, et al. (2013a).

3. Energy shaping of IPHS

In Ramirez, Gorrec, et al. (2013) the framework of the thermodynamic availability function, formalized for the control of thermodynamic systems by Alonso and Ydstie (2001) and with roots in the works of Keenan (1951) and Willems (1972), has been used to derive a Lyapunov condition for the stability analysis of IPHS. Using the convexity of the internal energy function, a convex extension named *energy based availability function* has been defined and shown to be a Lyapunov function candidate for the closed-loop system. This has been done following Alonso and Ydstie (2001), Hoang et al. (2011, 2012) and Ydstie (2002), where an entropy based availability function is constructed for irreversible thermodynamic systems.

In the sequel the stability condition presented in Ramirez, Gorrec, et al. (2013) is developed and it is shown that it defines an *energy shaping controller* (Ortega et al., 2001, 2002) with respect to a new closed-loop Hamiltonian and supply rate.

Definition 2. The *energy based availability function* is defined as

$$A(x, x^*) = U(x) + U_a(x, x^*) \tag{5}$$

with

$$U_a(x, x^*) = -U(x^*) - \frac{\partial U}{\partial x}(x^*)^\top (x - x^*) \tag{6}$$

and x^* an equilibrium point.

Assumption 3. The availability function $A(x, x^*)$ is strictly positive with minimum $A(x = x^*) = 0$ where x^* is an equilibrium point.

This assumption is fulfilled for any equilibrium point of a monophasic thermodynamic systems if one of the extensive variables is fixed since then the internal energy is a strictly convex function (Jillson & Ydstie, 2007).

It is clear from Definition 2 that the energy based availability function qualifies as a Lyapunov function candidate for controlled