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Brief paper

Nonsmooth leader-following formation control of nonidentical multi-agent systems with directed communication topologies[☆]

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ABSTRACT

This paper presents two nonsmooth leader-following formation protocols for nonidentical Lipschitz nonlinear multi-agent systems with directed communication network topologies. One protocol is used to achieve finite-time formation for first-order systems, and the other to achieve asymptotic formation for second-order systems. In these protocols, the states of all the agents, including the leader and the followers, are available only locally within their neighborhoods. Some sufficient conditions for reaching formations are derived for nonidentical nonlinear systems satisfying locally Lipschitz conditions. To prove the stability, a new nonsmooth Lyapunov function is constructed, with stability conditions derived under a nonsmooth analysis framework. The proposed formation protocols are applied to multi-spacecraft systems in deep-space exploration, with numerical simulations demonstrating the effectiveness of the theoretical results.

1. Introduction

Formation control is an important task in cooperative control of multi-agent systems due to its broad applications to unmanned aerial vehicles, autonomous spacecraft, autonomous underwater vehicles, automated military systems, sensor networks, biological systems, and so on. The objective of formation is to form and maintain a certain position-orientation pattern during the motion of a group of mobile agents (Chen, Wang, & Li, 2012); that is, the movement of all agents is to achieve and then maintain pre-specified relative positions and orientations with respect to each other. A problem closely related to formation control is synchronization, in which the goal of all agents is to achieve and then maintain the same state; therefore, it can be considered as a special case of formation.

Significant progress has been made in the study of formation control for multi-agent systems, where most works considered simple cases in which the agents are governed by first-order dynamics. To name a few, the synchronization problem for single-integrator multi-agent systems and linear multi-agent systems was considered in Cao, Ren, and Li (2009), Hui (2011), Li, Duan, Chen, and Huang (2010); Tuna (2009), and finite-time synchronization for nonlinear multi-agent systems was analyzed in Cao and Ren (2012), Li and Qu (2014) and Zhang, Yang, Zhao, and Wen (2013). In Zhang et al. (2013), the interactions between followers are required to be bidirectional. In Cao and Ren (2012), the interaction graph is directed, but the nonlinear functions are determined by the corresponding state components which, however, may not be satisfied in practice. In Li and Qu (2014), the interaction topologies are directed and switching, but the nonlinear systems are identical and bounded. In the past few years, some general formation control problems of first-order multi-agent systems were also investigated, for instance a finite-time formation control protocol was proposed for single-integrator multi-agent systems in Cao, Ren, and Meng (2010). Asymptotic formation control problems of multi-agent systems with linear dynamics were studied in Fax and Murray (2004) and Lafferriere, Williams, Caughman, and Veerman (2005).

Recently, the second-order formation problem has been recognized as an important topic for study. As a special case

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of formation, synchronization in double-integrator multi-agent systems was considered in Hong, Chen, and Bushnell (2008) and Ren (2008). Subsequently, for nonlinear multi-agent systems, synchronization was analyzed in Song, Cao, and Yu (2013) and Wen, Duan, Yu, and Chen (2013), where the nonlinear functions are identical. In Hu (2012), the nonlinear functions can be nonidentical but should have first-order and second-order derivatives. In Ding (2013) and Dong and Huang (2014), the nonidentical nonlinear functions are polynomials or smooth functions, and they have to be determined by the first and the next state components, where the tracking signals are generated by a linear system. In Liu and Jiang (2013), the nonlinear functions can be nonidentical and satisfy the Lipschitz conditions, but they are also determined by the first and the next state components, and moreover the states of all agents converge to a constant value. Some general formation control problems of second-order multi-agent systems were also investigated. For double-integrator multi-agent systems, the formation problem was investigated in Chen, Chen, Liu, Xiang, and Yuan (2008), Ren (2006), and for nonlinear multi-agent systems the formation problem was analyzed in Li, Chen, and Liu (2013) and Wang and Wu (2012). In Li et al. (2013), the nonlinear functions are identical and determined only by the velocity term. In Wang and Wu (2012), the nonlinear functions are identical and each agent needs to obtain information of the leader. However, in many practical cases, the agents can only share information with their neighbors, so the design of control protocols has to be based on local information through neighboring interactions.

Letting the pre-specified relative positions be zero, formation protocols will be reduced to synchronization protocols; however, the reverse may not be possible in general. So, two leader-following formation protocols are proposed in this paper: one is used to achieve finite-time formation for first-order systems and the other to achieve asymptotic formation for second-order systems. The main contribution of this paper is twofold: (i) The nonlinear functions of all agents, including the leader and the followers, can be nonidentical and only need to satisfy Lipschitz conditions. (ii) A new nonsmooth Lyapunov function is constructed to prove the stability of the overall network. Using the new Lyapunov function, the stability analysis can be performed under a nonsmooth analysis framework. Thus, the proof technique of this paper is different from the others used in the literature, and is more rigorous from a mathematical point of view.

The rest of the paper is organized as follows. Section 2 introduces some preliminaries. Section 3 discusses the finite-time first-order formation tracking problem with a directed communication topology and nonlinear agent dynamics. Section 4 discusses the asymptotic second-order formation tracking problem with a directed topology and nonlinear dynamics. Section 5 shows some examples in deep-space exploration to verify the theoretical results. Finally, Section 6 summarizes the investigation. To have better readability of this paper, most detailed proofs of the results are presented in a supplementary file (Lü, Chen, & Chen, 0000).

2. Background and preliminaries

Denote by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ a *weighted directed graph* of order N, with a set of nodes $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$, a set of directed edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \boldsymbol{R}^{N\times N}$. The node indices belong to a finite integer set $\mathcal{I} = \{1, 2, \dots, N\}$. A *directed edge* $\mathcal{E}_{ij}$ in the network $\mathcal{G}$ is denoted by the ordered pair of nodes (v_i, v_j), which means that node v_j can receive information from node v_i. The set of *neighbors* of node v_i is defined as $\mathcal{N}_{v_i} = \{v_j \subset \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$. The *adjacency matrix* of a weighted directed graph is defined as follows: $a_{ij} > 0$ if and only if $v_j \in \mathcal{N}_{v_i}$, otherwise $a_{ij} = 0$, and $a_{ii} = 0$ for all $i \in \mathcal{I}$. A *directed path* is an ordered sequence of edges $\{(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \dots, (v_{i_m-1}, v_{i_m})\}$, where $1 < i_m \le N$. In a directed graph, a *directed tree* consists of all directed paths without loops. A node v_i is called a *root of a directed tree* if for every node v_j in the graph there is a directed path connecting v_i to v_j. A *directed spanning tree* is a directed tree that connects all nodes of the network.

Next, some mathematical preliminaries on nonsmooth analysis are introduced. Consider a vector differential equation given by $\dot{z} = g(t, z)$, where $z \in \boldsymbol{R}^m$, and $g : \boldsymbol{R}^m \to \boldsymbol{R}^m$ is Lebesgue measurable and locally essentially bounded. A *Filippov solution* of the above differential equation is defined to be an absolutely continuous function $z : [0, \tau] \to \boldsymbol{R}^m$ such that $\dot{z}(t) \in \mathcal{K}[g](t, z)$, where $\mathcal{K}[g](t, z) = \bigcap_{\delta>0} \bigcap_{\mu(S)=0} \overline{co}\{g(\mathcal{B}_\delta(z) \setminus S)\}$, $\mu(.)$ denotes the usual Lebesgue measure in $\boldsymbol{R}^m$, $\overline{co}$ denotes the convex closure, $\mathcal{B}_\delta(z)$ denotes the open ball of radius δ centered at z, and S denotes the set of zero measure. A function $f : \boldsymbol{R}^d \to \boldsymbol{R}^m$ is *locally Lipschitz at* $x \in \boldsymbol{R}^d$ if there exists a constant L_x such that $\|f(t, y)-f(t, y')\| \le L_x\|y - y'\|$ for all $y, y' \in B(x, \epsilon)$, where $B(x, \epsilon)$ is a ball of radius $\epsilon > 0$ centered at x. A function is *locally Lipschitz on a region* $J \subset \boldsymbol{R}^d$ if it is locally Lipschitz at x for all $x \in J$, and if $J = \boldsymbol{R}^d$ then it is global. Given $f : \boldsymbol{R}^d \to \boldsymbol{R}$, the *right directional derivative* of f at x in the direction $v \in \boldsymbol{R}^d$ is defined as $f'(x; v) = \lim_{h\to 0^+} \frac{f(x+hv)-f(x)}{h}$ when this limit exists. The *generalized directional derivative* of f at x in the direction $v \in \boldsymbol{R}^d$ is defined as $f^o(x; v) = \limsup_{\substack{y\to x \\ h\to 0^+}} \frac{f(y+hv)-f(y)}{h}$ when this limit exists. A function $f : \boldsymbol{R}^d \to \boldsymbol{R}$ is *regular* at $x \in \boldsymbol{R}^d$ if $f'(x; v) = f^o(x; v)$ for all $v \in \boldsymbol{R}^d$. Given a locally Lipschitz function $f : \boldsymbol{R}^d \to \boldsymbol{R}$ and a set-valued map $\mathcal{F} : \boldsymbol{R}^d \to \mathcal{B}(\boldsymbol{R}^d)$, where $\mathcal{B}(\boldsymbol{R}^d)$ is the set consisting of all possible subsets of $\boldsymbol{R}^d$, the *set-valued Lie derivative* $\tilde{\mathcal{L}}_F f : \boldsymbol{R}^d \to \mathcal{B}(\boldsymbol{R}^d)$ of f with respect to $\mathcal{F}$ at x is defined as $\tilde{\mathcal{L}}_F f(x) = \{a \in \boldsymbol{R} :$ there exists $v \in \mathcal{F}(x)$ such that $\zeta^T v = a$ for all $\zeta \in \partial f(x)\}$, where ∂f is the general gradient of the locally Lipschitz function f.

In addition, the following lemma is needed to derive the main results of the paper.

Lemma 1 (Cortés, 2008a). *Let $\mathcal{F} : \boldsymbol{R}^d \to \mathcal{B}(\boldsymbol{R}^d)$ be a set-valued map, x_e be an equilibrium of $\mathcal{F}$, and $\mathcal{D} \subseteq \boldsymbol{R}^d$ be an open connected set with $x_e \in \mathcal{D}$. Furthermore, let $f : \boldsymbol{R}^d \to \boldsymbol{R}$ satisfy the following conditions:*

(1) *f is locally Lipschitz and regular on $\mathcal{D}$.*
(2) *$f(x_e) = 0$, and $f(x) > 0$ for all $x \in \mathcal{D} \setminus \{x_e\}$.*
(3) *$\max \tilde{\mathcal{L}}_F f(x) < 0$ for all $x \in \mathcal{D} \setminus \{x_e\}$.*

Then, x_e is a strongly asymptotically stable equilibrium, in the sense that the asymptotical stability is retained by all the solutions in the set-valued map $\mathcal{F}$.

3. Distributed formation control of first-order nonlinear multi-agent systems

In this section, the finite-time formation control problem for first-order nonlinear multi-agent systems with a directed communication network topology is studied. The basic idea is to design the control laws strong enough to attenuate the effect of the nonidentical and nonlinear system dynamics in finite time.

3.1. Problem description

Consider a first-order multi-agent system consisting of N follower agents and one leader agent. The dynamics of the followers and the leader are described by

$$\begin{aligned} \dot{x}_i(t) &= f_i(t, x_i(t)) + u_i(t), \quad i = 1, 2, \dots, N, \\ \dot{x}_0(t) &= f_0(t, x_0(t)), \end{aligned} \tag{1}$$

respectively, where $f_i : \boldsymbol{R} \times \boldsymbol{R}^n \to \boldsymbol{R}^n$ describes the nonlinear dynamics of follower i, $f_0 : \boldsymbol{R} \times \boldsymbol{R}^n \to \boldsymbol{R}^n$ is the nonlinear dynamics of the leader, $x_i \in \boldsymbol{R}^n$ and $x_0 \in \boldsymbol{R}^n$ are the states of follower i and the leader respectively, and $u_i \in \boldsymbol{R}^n$ is the control input, $i = 1, 2, \dots, N$.