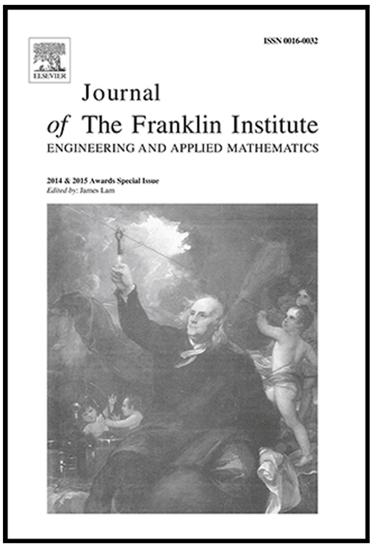

An accelerated technique for solving the positive definite solutions of a class of nonlinear matrix equations

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An accelerated technique for solving the positive definite solutions of a class of nonlinear matrix equations

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Abstract

Nonlinear matrix equations (NMEs) are encountered in many applications of control and engineering problems. In this work, we establish a complete study for a class of nonlinear matrix equations. With the aid of Sherman Morrison Woodbury formula, we have shown that any equation in this class has the maximal positive definite solution under certain conditions. Furthermore, a thorough study of properties about this class of matrix equations is provided. An acceleration of iterative method with R-superlinear convergence is then designed to solve the maximal positive definite solution. Two numerical experiments demonstrate that our methods perform efficiently and reliably.

Keywords: Nonlinear matrix equation, Sherman Morrison Woodbury formula, Maximal positive definite solution, Semigroup property, Positive operator, Doubling algorithm, R-superlinear with order r
2000 MSC: 39B12, 39B42, 65H05, 15A24,

1. Introduction

Nonlinear matrix equations (NMEs) $X+A^HX^{-1}A=Q$ and $X-A^HX^{-1}A=Q$ occur frequently in many applications. For example, the applications of first equation include control theory, ladder networks, dynamic programming, stochastic filtering and statics [1, 27]. Other notable examples include algebraic Riccati equations [10, 9, 18, 17, 22]. The second equation appears in a problem in the analysis of stationary Gaussian reciprocal processes over a finite interval [19, 23, 15]. Recently, a kind of nonlinear matrix equations $X + A^H\bar{X}^{-1}A = Q$ has received considerable attention. The main application of this equation arises from the study of modern quantum theory by means of consimilarity. Some

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