Automatica 64 (2016) 133–142

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Observer-based adaptive sliding mode control for nonlinear Markovian jump systems ☆

Hongyi Li [a,1], Peng Shi [b,c,d], Deyin Yao [a], Ligang Wu [e]

[a] College of Engineering, Bohai University, Jinzhou 121013, Liaoning, China
[b] School of Electrical and Electronic Engineering, University of Adelaide, Adelaide, SA 5005, Australia
[c] College of Engineering and Science, Victoria University, Melbourne VIC 8001, Australia
[d] College of Automation, Harbin Engineering University, Harbin 150001, China
[e] Research Institute of Intelligent Control and Systems, Harbin Institute of Technology, Harbin 150001, Heilongjiang, China

ARTICLE INFO

Article history:
Received 9 November 2014
Received in revised form
26 August 2015
Accepted 19 October 2015
Available online 7 December 2015

Keywords:
Markovian jump systems (MJSs)
Adaptive sliding mode control
Partly unknown transition probabilities

ABSTRACT

This paper investigates the adaptive sliding mode control problem of nonlinear Markovian jump systems (MJSs) with partly unknown transition probabilities. The system state components are not all unmeasured. The specific information of the model uncertainties and bounds of the nonlinear term and disturbance term are unknown in the controller design process. Moreover, any knowledge of the unknown elements existing in the transition matrix is not required. Our attention is mainly focused on designing the observer-based adaptive sliding mode controller for such a complex system. Firstly, an observer is constructed to estimate the system state. Secondly, we design an integral sliding mode surface and observer-based adaptive sliding mode controller such that the MJSs are insensitive to all admissible uncertainties and satisfy the reaching condition. The stochastic stability of the closed-loop system can be guaranteed. Finally, a numerical example is exploited to demonstrate the effectiveness of the proposed results.

1. Introduction

It is well known that many dynamical systems may subject to a few unpredictable structural changes, such as random failures, repairs of sudden environment disturbances, etc. (Wu, Yao, & Zheng, 2012). Markovian jump systems (MJSs) are a special type of Markovian switching systems. The mode switching (Mao, 2002) is dominated under a Markov process. Many problems of stability, stabilization, and filtering for MJSs have been investigated (Chen & Zheng, 2015; Karimi, 2011; Li, Gao, Shi, & Zhao, 2014; Niu, Ho, & Wang, 2007; Wang, Liu, & Liu, 2010; Wang, Qiao, & Burnham, 2002; Xia, Fu, Shi, Wu, & Zhang, 2009). In Li et al. (2014), the authors proposed a novel augmented sliding mode observer scheme in solving the stabilization problem for a type of Markovian stochastic jump systems. In Wang et al. (2010), the authors proposed a novel exponential stabilization method for stochastic MJSs with mixed time-delays. The authors studied the problem of sliding mode control (SMC) for a type of nonlinear uncertain stochastic systems with Markovian switching in Niu et al. (2007).

SMC is an effective robust control algorithm since it is insensitive to model uncertainties, external disturbances and parameter variations. In the past few decades, the SMC approach has been employed to a large number of physical systems such as robot manipulators, automotive engines and power systems. Therefore, the SMC design problem has received considerable attention and there are some significant results for linear or nonlinear systems (Basin, Ferreira, & Fridman, 2007; Basin, Fridman, Rodriguez-González, & Acosta, 2003; Basin & Rodriguez-Ramirez, 2012; Davila, Fridman, & Levant, 2005; Edwards, Spurgeon, & Patton, 2000; Feng, Yu, & Man, 2002; Hu, Wang, Gao, & Stergioulas, 2012; Liu, Shi, Zhang, & Zhao, 2011; Niu & Ho, 2010; Niu, Ho, & Lam, 2005; Roh & Oh, 1999; Shtessel, Shkolnikov, & Levant, 2007; Xia & Jia, 2002; Young, Utkin, & Ozguner, 1999). In Basin and Rodriguez-Ramirez (2012), the authors designed a sliding mode mean-square filter of nonlinear polynomial systems with unmeasured states. In Niu et al. (2005), the authors developed a novel SMC approach for uncertain stochastic systems with time delay.

However, the system states are not always known in practice. It should be pointed out that the inevitable uncertainties

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Michael V. Basin under the direction of Editor Ian R. Petersen.

E-mail addresses: lihongyi2009@gmail.com (H. Li), peng.shi@adelaide.edu.au (P. Shi), deyinyao2013@gmail.com (D. Yao), ligangwu@hit.edu.cn (L. Wu).

[1] Tel.: +86 416 3719182.

http://dx.doi.org/10.1016/j.automatica.2015.11.007

including modeling error, parameter perturbations and external disturbances may appear in real systems. The authors in Wu, Shi, and Gao (2010) used state estimation and SMC method to control a series of Markovian jump singular systems with unmeasured states. The SMC method was proposed to stabilize Markovian jump linear systems with disturbances in Shi, Xia, Liu, and Rees (2006). The authors in Zhang and Boukas (2009) investigated the stability and stabilization of Markovian jump linear systems with partly unknown transition probabilities. It is noted that all the approaches mentioned above did not consider the issues of SMC and state estimation for MJSs with partly unknown transition probabilities, matched nonlinearity, model uncertainties and matched disturbance. Therefore, the problems of SMC and state estimation for MJSs with partly unknown transition probabilities, matched nonlinearity, model uncertainties and matched disturbance still remain open due to some difficulties. First, it is very difficult to handle the unknown matched nonlinearity which is different from the traditional Lipschitz condition. Second, the exact information of the bounds of nonlinearity and external disturbance is unknown, which may lead to the instability of the closed-loop system. Third, it is quite difficult to measure the system states in practical control systems. Fourth, all of the elements in the transition probability matrix are hard to be obtained. These four aspects motivate our current research work of this paper.

In this paper, we consider the adaptive SMC problem for MJSs with partly unknown transition probabilities, matched nonlinearity, matched disturbance, model uncertainties and unmeasured states. The model uncertainty satisfies norm-bounded and the specific information involving bounds of the nonlinear term and disturbance term is unknown. The partly unknown transition probabilities proposed in this paper not require any knowledge of the unknown elements. Firstly, an appropriate integral sliding mode surface is constructed such that the reduced-order equivalent sliding motion can adjust the effect of the chattering phenomenon in the plant. Secondly, the observer-based adaptive sliding mode controller is designed to adapt the unknown upper bounds of matched nonlinearity and disturbance and guarantee the stochastic stability of the closed-loop system. Finally, a numerical example is provided to show the effectiveness of the proposed scheme.

The organization of this paper is given as follows. The system description and some preliminaries are given in Section 2. Section 3 presents the design of an integral sliding mode surface function and Section 4 designs an adaptive sliding mode controller. Section 5 provides a numerical example to certify the feasibility of the mentioned method, and we conclude this paper in Section 6.

Notations: The superscript "T" represents the matrix transposition, $\mathbb{R}^n$ shows the n-dimensional Euclidean space. I and 0 denote the identity matrix and zero matrix, respectively. The notation $X(i) > 0$ means that $X(i)$ is real symmetric and positive definite. $\|\cdot\|_1$ and $\|\cdot\|_2$ refer to the 1-norm and usual Euclidean vector norm, respectively. $\lambda_{\max}(P)$ stands for the maximum eigenvalue of a real symmetric matrix P. The notation $\mathrm{diag}\{\cdot\}$ represents a diagonal matrix. The notation $(\Omega, \mathcal{F}, \mathcal{P})$ denotes the probability space. Ω, $\mathcal{F}$ and $\mathcal{P}$ represent the sample space, σ-algebra of subsets of the sample space and probability measure on $\mathcal{F}$, respectively. The notation $\mathbf{E}\{\cdot\}$ denotes the mathematical expectation. $\mathbf{P}\{\cdot\}$ represents the probability. The symbol "$*$" denotes a term that is induced by symmetry.

2. System description and preliminaries

The following MJSs are defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$\begin{cases} \dot{x}(t) = [A(r_t) + \triangle A(r_t, t)]\,x(t) \\ \qquad + B(r_t)\,[u(t) + f(x,t) + d(t)], \\ y(t) = C(r_t)\,x(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, f(x,t) \in \mathbb{R}^m, d(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$ denote the state vector, control input, nonlinear function, disturbance and measured output, respectively. $A(r_t) \in \mathbb{R}^{n\times n}, B(r_t) \in \mathbb{R}^{n\times m}$ and $C(r_t) \in \mathbb{R}^{p\times n}$ are constant system matrices with appropriate dimensions. $\triangle A(r_t, t) \in \mathbb{R}^{n\times n}$ is the system model uncertainty with appropriate dimensions. $\{r_t, t \geq 0\}$ is a finite-state Markov jumping process representing the system mode, which takes discrete values in a given state space $S = \{1, 2, \ldots, s\}$. Let $\Pi = (\pi_{ij})_{s\times s}$ $(i, j = 1, 2, \ldots, s)$ represent the transition rate matrix. Then the mode transition probabilities can be denoted as follows:

$$\Pr\left(r_{t+\triangle t} = j \mid r_t = i\right) = \begin{cases} \pi_{ij}\triangle t + o(\triangle t), & i \neq j, \\ 1 + \pi_{ii}\triangle t + o(\triangle t), & i = j, \end{cases}$$

where $\triangle t > 0$ and $\lim_{\triangle t\to 0} o(\triangle t)/\triangle t = 0$, π_{ij} satisfies $\pi_{ij} > 0$ with $i \neq j$ and $\pi_{ii} = -\sum_{j=1, j\neq i}^{s} \pi_{ij}$ for each mode i.

In addition, the transition rates or probabilities of the jumping process are assumed to be partly accessed, in other words, some elements in matrix Π are unknown. For example, there are four operation modes in system (1), the transition rates or probability matrix Π may be described as:

$$\Pi = \begin{bmatrix} ? & \pi_{12} & ? & ? \\ ? & \pi_{22} & \pi_{23} & ? \\ \pi_{31} & ? & \pi_{33} & ? \\ \pi_{41} & ? & ? & \pi_{44} \end{bmatrix}$$

where "?" denotes the inaccessible elements. For notational convenience, $\forall i \in S$, we represent $S = S_{\kappa}^i \cup S_{u\kappa}^i$ with

$$S_{\kappa}^i \triangleq \left\{ j : \pi_{ij} \text{ is known} \right\}, \qquad S_{u\kappa}^i \triangleq \left\{ j : \pi_{ij} \text{ is unknown} \right\}.$$

We represent $\pi_{\kappa}^i \triangleq \sum_{j\in S_{\kappa}^i} \pi_{ij}$ in the paper. For notional convenience, when the system operates in the ith mode, MJSs (1) can be rewritten as

$$\begin{cases} \dot{x}(t) = [A(i) + \triangle A(i,t)]\,x(t) \\ \qquad + B(i)\,[u(t) + f(x,t) + d(t)], \\ y(t) = C(i)\,x(t), \end{cases} \tag{2}$$

where $A(r_t) = A(i)$, $B(r_t) = B(i)$ and $C(r_t) = C(i)$. $\triangle A(r_t, t) = \triangle A(i, t)$ denotes the system model uncertainty and satisfies the following form:

$$\triangle A(i,t) = D(i)\,F(i,t)E(i), \tag{3}$$

where $D(i)$ and $E(i)$ are the constant matrices with appropriate dimensions, and $F(i,t)$ is an unknown time-varying matrix function satisfying

$$F^T(i,t)F(i,t) \leq I, \quad i \in S. \tag{4}$$

The external disturbance vector $d(t)$ is an unknown function. The matched disturbance $d(t)$ is bounded as $\|d(t)\| \leq d$, where d is an unknown scalar. The nonlinear function $f(x,t)$ satisfies

$$\|f(x,t)\| \leq \alpha + \beta\|y(t)\|, \tag{5}$$

where $\alpha > 0$ and $\beta > 0$ are unknown constants. In the paper, similar to the existing results (Niu et al., 2007; Wu et al., 2010), it is assumed that the input matrix $B(i)$ has full column rank for each $i \in S$.

The following definition is essential for this paper.

Definition 1 (*Chen, Niu, & Zou, 2013*)**.** The equilibrium solution, $x_t = 0$, of the MJSs (2) with $u(t) = 0$ is said to be globally asymptotically stable (with probability one) if for any $h \geq 0$ and $\varsigma > 0$,

$$\lim_{\mathbf{x}\to\mathbf{0}} \mathbf{P}\left\{\sup_{\mathbf{h}<\mathbf{t}} \left|x_t^{h,x}\right| > \varsigma\right\} = \mathbf{0}, \qquad \mathbf{P}\left\{\lim_{\mathbf{t}\to+\infty} \left|x_t^{h,x}\right| = 0\right\} = \mathbf{1}, \tag{6}$$

where $x_t^{h,x}$ denotes the solution at time t of MJSs starting from the state x at time h for $h \leq t$.