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Dissipative solitons in forced cyclic and symmetric structures

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ABSTRACT

The emergence of localised vibrations in cyclic and symmetric rotating structures, such as bladed disks of aircraft engines, has challenged engineers in the past few decades. In the linear regime, localised states may arise due to a lack of symmetry, as for example induced by inhomogeneities. However, when structures deviate from the linear behaviour, e.g. due to material nonlinearities, geometric nonlinearities like large deformations, or other nonlinear elements like joints or friction interfaces, localised states may arise even in perfectly symmetric structures. In this paper, a system consisting of coupled Duffing oscillators with linear viscous damping is subjected to external travelling wave forcing. The system may be considered a minimal model for bladed disks in turbomachinery operating in the nonlinear regime, where such excitation may arise due to imbalance or aerodynamic excitation. We demonstrate that near the resonance, in this non-conservative regime, localised vibration states bifurcate from the travelling waves. Complex bifurcation diagrams result, comprising stable and unstable dissipative solitons. The localised solutions can also be continued numerically to a conservative limit, where solitons bifurcate from the backbone curves of the travelling waves at finite amplitudes.

1. Introduction

Localisation of vibrations in cyclic and symmetric structures is an important topic for rotating machines due to high cycle fatigue. In the case of linear vibrations, localisation may occur due to a lack of symmetry, induced e.g. by manufacturing variabilities or wear [1–3]. This problem, usually referred to as a mistuning problem, has attracted much attention in the literature, encompassing areas such as efficient numerical tools for analysis [4], experimental investigations [5], and even the use of intentional mistuning to minimise localised vibrations [6].

In real engineering applications, the assumption of linear vibrations has to be seen as an approximation, and the dynamics of complex engineering structures is usually a result of multiple linear and nonlinear phenomena. In the case of bladed disks of aircraft engines, nonlinearities may arise e.g. due to friction induced by internal joints [7,8], or due to large deformations induced by strong excitations [9]. In the nonlinear regime, the processes which lead to localised vibrations may go beyond

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mistuning. It is well-known, for example, that even perfect cyclic and symmetric structures may localise vibrations due to bifurcations [10–15]. Moreover, the effects of nonlinearities in nonhomogeneous systems is still a very active topic of research not only in structural dynamics [16], but also in other research fields [17–19].

This paper focuses on the nonlinear dynamics of cyclic and symmetric structures that are excited by forces that have the form of travelling waves. In the case of bladed disks, such excitations, also known as engine orders, are caused by aerodynamic or structural forces that move along the structure preserving their shape. Analysis is based on a minimal model composed of symmetric Duffing oscillators, cyclically connected to each other, and in the presence of linear velocity-proportional dampers. It has already been shown that, in the conservative regime, the underlying model has localised states composed of envelope solitons [20,21]. These results were obtained from analytical solutions of a Nonlinear Schrödinger Equation (NLSE). For the non-conservative case, here a modified NLSE is developed, and analytical solutions describing localised states are not possible anymore. Therefore, general solutions and their stability are assessed numerically. Within this regime, it is demonstrated that localised states, sometimes called dissipative solitons [22,23], branch off from the travelling waves, resulting in complex bifurcation diagrams composed of stable and unstable single and multi-soliton states. Finally, these localised states are continued into the conservative limit and it is demonstrated that they become nonlinear normal modes of the underlying system. These modes are forced to bifurcate from the backbone curve of the travelling wave at finite vibration amplitude.

The paper is organized as follows. In Section 2 the minimal model for weakly nonlinear cyclic and symmetric structures is introduced. Moreover, the assumptions necessary to derive a non-conservative NLSE are discussed, and strategies to compute localised solutions and check their stability are presented. General numerical results are shown in Section 3. We discuss how localised states branch off from travelling waves in the NLSE framework, and these results are verified through time integration of the full equations. In Section 4, localised states obtained from the non-conservative NLSE are continued into the conservative limit. Bifurcation diagrams show that localised states detach from travelling waves at non-zero amplitudes, contrary to the standard analytical predictions obtained from the conservative NLSE solved in an infinite spatial domain. Finally, Section 5 discusses the main conclusions and suggests directions for further investigations.

2. The minimal model and solution methods

The physical system under investigation is displayed in Fig. 1. It consists of N_s blocks of mass m, cyclically connected through linear springs k_c, and attached to the ground by linear and cubic springs k_l and k_{nl}, respectively. The dissipation is modelled by linear viscous dampers c, connected to the ground, while the external force applied to the nth mass is f_n. The system in Fig. 1 can be understood as a minimal model of a bladed disk [9,24] operating in the strong deformation regime, while a positive value of k_{nl} models the induced hardening effect.

The displacement of the nth degree of freedom is given by

$$m\ddot{u}_n + c\dot{u}_n + k_l u_n - k_c(u_{n-1} + u_{n+1} - 2u_n) + k_{nl}u_n^3 = f_n. \tag{1}$$

This equation can be written in rescaled variables, for convenience, as

$$\ddot{u}_n + \gamma^2\dot{u}_n + \omega_0^2 u_n - \omega_c^2(u_{n-1} + u_{n+1} - 2u_n) + \xi u_n^3 = g_n, \tag{2}$$

where $\gamma^2 = c/m, \omega_0^2 = k_l/m, \omega_c^2 = k_c/m, \xi = k_{nl}/m$ and $g_n = f_n/m$. In this paper, the investigation focuses on responses to travelling wave excitation. The system dynamics induced by these forces is a typical analysis in bladed disks vibrations. Travelling wave excitation is induced in rotating machines e.g. due to aerodynamic forces that move along the structure [25] or due to engine orders [26]. Mathematically, a travelling wave excitation can be written as

$$g_n(t) = F_0 \exp\{\mathrm{i}[k(n-1)a - \omega_f t]\} + \text{c.c.}, \tag{3}$$

where F_0 is the force amplitude, k is its wave number, $a = 2\pi/N_s$ is the lattice parameter, ω_f is the external frequency, i is the imaginary unity, and c.c. represents the complex-conjugate of the preceding expression. It should be noted that the wave number k can only assume integer numbers and, consequently, the external force respects the cyclic symmetry ($g_{N_s+1} = g_1$).

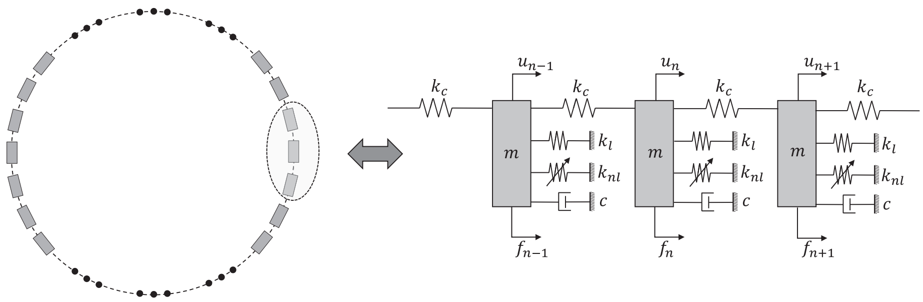

Fig. 1. Physical system under investigation. In the left-hand side, an illustration of the full system, while the right-hand side displays any three neighbouring degrees of freedom.