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Development of a theoretical framework for vibration analysis of the class of problems described by fractional derivatives

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ABSTRACT

Fractional derivative is increasingly being deployed to improve existing mathematical models due to its unique capability in describing anomalous behavior and memory effects which are common characteristics of natural phenomena. Improved vibration analysis has been accomplished by introducing fractional derivatives to model viscoelastic damping and vibration propagation through complex media and much research has been carried out to date. However, much of these existing research efforts have been sporadic to the best and there remains a pressing need to develop a consistent and systematic theoretical framework for vibration analysis of fractional systems to synergize for more productive and coordinated efforts in the area. This paper seeks to address some fundamental issues to facilitate further development such as the definition of general form of a fractional vibration system, its eigenvalue problem and methods of solution, definition of frequency response functions and applicability of conventional modal analysis, equivalent eigensystem and its more efficient eigensolution. With these important issues being resolved to clear the myth, vibration studies of fractional systems can be encouraged and expected to grow in a more fruitful direction. New methods developed and concepts discussed in the paper have all been validated through realistic numerical case studies based on a practical GARTEUR structure with viscoelastic supports modeled using fractional derivatives.

1. Introduction

The general concept of fractional derivative dates back to the beginning of differential calculus when Leibniz discussed fractional derivatives in his letter to L'Hôpital in 1695 [1]. Subsequently, the mathematical foundation of fractional derivatives was laid through combined efforts of great mathematicians such as Euler, Liouville, Riemann, Letnikov and Caputo [2]. However, only until recently that important applications of fractional derivatives to various branches of science and engineering have been established. It has become certain nowadays that one will find interesting and novel important applications of fractional calculus to mathematics, physics, chemistry, biology, engineering, finances, psychology and other emerging areas that have been rigorously developed in the last few decades.

Abel's study of the tautochrone problem [3] is perhaps the first earliest application of fractional derivatives to an engineering problem by establishing the paths in which the time taken for an object to fall under the gravity becomes independent of initial starting positions. Fractional conservation of mass equations have been developed to better model fluid flow

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when control volume chosen is not large enough compared to the scale of heterogeneity and the flux within the control volume is considered to be nonlinear [4]. In the modelling of groundwater flow problem, Antangana et al. [5,6] used fractional derivatives to generalize the Darcy law and combined it with conservation of mass to develop a new equation governing the flow of groundwater. Fractional advection dispersion equation has been used to model contaminant flow in heterogeneous porous media and the results have shown more reliable predictions of pollutions than the existing integer ordered equations. Anomalous diffusion process in complex media has been found to be better characterized using fractional-order diffusion equations [7] in which fractional time derivative corresponds to long-time heavy tail decay and fractional spatial derivative diffusion nonlocality. In fluid mechanics, the application of fractional calculus to the classical transient viscous diffusion equation in a semi-infinite space has been studied and the fractional methodology has been validated and shown to be much simpler and more powerful than existing techniques [8,9]. For vibration propagation through complex media such as biological tissues, better prediction of power flow has been accomplished through the introduction of fractional derivatives [10,11]. To increase the degrees of freedom to achieve better optimal control, fractional PID controllers were examined and discussed, demonstrating potential advantages over conventional controller designs [12]. In signal processing, interpolation functions were derived [13] which were necessary for the direct evaluations of fractional derivatives from raw sample data values. A new class of fractional-order anisotropic diffusion equations were introduced for noise removal associated with image processing [14]. A fractal scaling model which results in a fractional time derivative was used to predict the relaxation processes and reaction kinetics of proteins [15]. To model a combustion process, a fractional integro-differential equation was employed for modeling propagation of spherical flame [16]. Hausdorff fractal distance concept was developed and applied to solve the spatially fractional partial differential equations capable of modeling physical characteristics such as anomalous diffusion, creep and relaxation in fractal media [17]. In finance engineering, fractional diffusion equation was used to model the continuous time behavior of the dynamics of financial markets [18]. And even in human psychology, fractional order derivatives have found applications in the modelling of complex human behavior such as love [19] and happiness [20].

Improved vibration analysis has been accomplished by introducing fractional derivatives to model viscoelastic damping and vibration propagation through complex media and much research has been conducted during the last 2 decades. Free damped vibrations of an oscillator, whose viscoelastic properties are described in terms of the fractional Kelvin-Voight and Maxwell models, were analyzed to establish system stability under various system parameters [21]. Random vibration behavior of a class of models that describe dynamic systems with frequency dependent fractional derivatives was investigated and standard formula was developed to predict vibration responses of such systems under random excitations [22]. Transversal oscillations of layered beams of nonhomogeneous and continuously creeping material with fractional modulus of elasticity were analyzed [23]. Damped vibrations of rectangular plates with viscoelastic damping modelled as fractional derivatives were examined and the effect of viscosity on overall vibration response was studied [24]. Dynamic response of plane inhomogeneous anisotropic bodies made of linear viscoelastic material described by differential constitutive equations with fractional order derivatives was investigated and effective solution procedures were developed [25]. Vibration of a rectangular plate with Pasternak-type viscoelastic foundation support modeled using fractional derivatives to characterize its time dependent characteristics has been undertaken [26]. Linear vibrations of axially moving systems which are modelled by a fractional derivative were formulated and approximate analytical solution was established by applying the method of multiple scales [27]. Vibration analysis method of magneto-thermo-viscoelasticity with diffusion based on fractional order model was developed and applied to a 2D boundary value problem [28]. For a taut cable with a viscous damper at arbitrary locations and portrayed by a five-parameter fractional derivative model, accurate vibration prediction was achieved over a wide frequency range [29]. To actively control vibrations of fractional systems, fractional order controller and feedback controller designs have been developed [30,31]. For passive vibration controls on the other hand, a novel tuned liquid column damper was developed and its damping characteristics were accurately modeled using fractional derivative approach [32,33]. Frequency response analysis of fractional MDOF vibration systems has gained much research interest recently. Effective methods have been developed which can be applied to predict vibration responses of fractional MDOF system in both time and frequency domains [34,35]. For a Euler Bernoulli beam subjected to fractional viscoelastic damping, vibration response analysis has been undertaken [36]. Stochastic vibration responses of fractionally damped beams have also been derived [37]. A new method was discussed which can be applied to solve dynamic problems of fractional derivative viscoelasticity [38]. An augmented state-space method was developed to analyze vibration response of MDOF system with fractional derivative element [39]. Application of fractional derivatives to seismic vibration analysis of base-isolated models was discussed in [40] and an finite element procedure was developed to analyze vibration responses of a cantilevered beam with fractional Kelvin-Voigt viscoelastic damping model [41]. Further, vibration analysis of fractional systems at micro/nano scales has gained increasing interests over recent years [42,43].

Alongside with these studies, research on nonlinear vibration analysis of fractional systems has also gained much momentum recently. Vibration transmission of a single degree of freedom oscillator with nonlinear fractional order damping was examined by Jian et al. [44]. Using the method of multiple scales, nonlinear vibration analysis of a 2DOF mechanical system with fractional damping was conducted and in the cases of one-to-one or two-to-one internal resonances, soliton-like solutions have been analytically established [45]. An efficient Adomian decomposition method was applied to a fractionally damped mechanical oscillator to derive an analytical solution which proves to be efficient for the calculations of transient vibration responses of fractional vibration systems [46]. Nonlinear random vibrations of a beam comprising a fractional derivative element and nonlinear terms due to large displacements were examined and the results showed that the