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The role of the Bhattacharyya distance in stochastic model updating

Sifeng Bi [a,*], Matteo Broggi [a], Michael Beer [a,b,c]

[a] Leibniz Universität Hannover, Institute for Risk and Reliability, Callinstrasse 34, Hannover, Germany
[b] University of Liverpool, Institute for Risk and Uncertainty, Peach Street, L69 7ZF Liverpool, United Kingdom
[c] Tongji University, International Joint Research Center for Engineering Reliability and Stochastic Mechanics, Shanghai 200092, China

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A B S T R A C T

The Bhattacharyya distance is a stochastic measurement between two samples and taking into account their probability distributions. The objective of this work is to further generalize the application of the Bhattacharyya distance as a novel uncertainty quantification metric by developing an approximate Bayesian computation model updating framework, in which the Bhattacharyya distance is fully embedded. The Bhattacharyya distance between sample sets is evaluated via a binning algorithm. And then the approximate likelihood function built upon the concept of the distance is developed in a two-step Bayesian updating framework, where the Euclidian and Bhattacharyya distances are utilized in the first and second steps, respectively. The performance of the proposed procedure is demonstrated with two exemplary applications, a simulated mass-spring example and a quite challenging benchmark problem for uncertainty treatment. These examples demonstrate a gain in quality of the stochastic updating by utilizing the superior features of the Bhattacharyya distance, representing a convenient, efficient, and capable metric for stochastic model updating and uncertainty characterization.

1. Introduction

It has been widely accepted that uncertainties should be appropriately considered in the campaign of model updating and validation. Uncertainty quantification (UQ) metrics are consequently significant to provide an elaborate measurement of the uncertainty information in stochastic model updating methodologies.

In the context of UQ, the system parameters can be categorized according to the involvement of aleatoric or/and epistemic uncertainties [1,2]:

I) parameters without any epistemic uncertainty, appearing as either explicit constants or random variables with fully determined uncertainty characteristics such as distribution type, mean, variance;
II) parameters with only epistemic uncertainty, modeled as constants but with unknown exact value bounded by a given interval;
III) parameters with both aleatoric and epistemic uncertainties, modeled as random variables with only vaguely determined uncertainty characteristics.

* Corresponding author.
E-mail address: sifeng.bi@irz.uni-hannover.de (S. Bi).

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As Category I parameters contain no epistemic uncertainties, they are investigated by neither deterministic nor stochastic model updating techniques. The deterministic updating methodologies generally investigate only Category II parameters aiming at a single set of crisp parameter values and at generating a single model prediction with maximum fidelity with regard to the observation. Comparatively, both Categories II and III parameters are considered by stochastic updating methodologies, whose target is not the single set of the parameters themselves, but a reduced space of the epistemic uncertainties, e.g. reduced intervals of Category II parameters and reduced bounds of the Cumulative Probability Function (CDF) of Category III parameters.

A wide range of stochastic model updating methodologies has been investigated in the literature, such as perturbation methods [3,4], interval updating [5,6] and fuzzy theory [7,8]. No matter which methodology is performed in stochastic model updating, it is significant to define a comprehensive UQ metric which is capable of quantifying the uncertain discrepancy between two random data samples, e.g. the observed data and the numerical model predictions.

As an alternative to the classical Euclidian distance, Khodaparast et al. [9] propose to use the Frobenius norm to measure the difference between the covariance matrices of the experimental and simulated data. By minimizing the objective function combining the Euclidian distance between the means and the Frobenius norm between the variances, this approach achieves satisfied updating result for both mean and variance with high efficiency. Another stochastic distance concept is the Bhattacharyya distance [10], which has been proposed as a potential UQ metric capable of capturing a higher level of statistical information from the investigating variables [11]. This metric is clearly more comprehensive for uncertainty treatment, however, its application in stochastic model updating is quite limited in the current literature. This limited application is not only caused by a relative high calculation cost but also, more critically, caused by its stochastic feature when integrating the product of the probability density functions (PDFs) of the random variables (which will be further discussed in Section 2.1). The objective of this work is consequently to generalize the Bhattacharyya distance as a universal metric for uncertainty treatment by developing a stochastic model updating approach, in which the Bhattacharyya distance is fully embedded and actively operated.

Bi et al. [11] performed a comprehensive comparison among the Euclidian, Mahalanobis, and Bhattacharyya distances as updating metrics via a direct Monte Carlo approach. However, this approach is essentially based on a single and non-directional searching technique with low efficiency. In the case of complex problems with high-dimensional parameter space, this approach would fail to search the global solutions. To solve this issue, in this paper, the Bayes' theorem [1] and transitional Markov chain Monte Carlo (TMCMC) algorithm [12] are employed to construct a Bayesian updating framework, in which the Bhattacharyya distance is embedded as the metric. In this framework, the approximate Bayesian computation (ABC) [13] is proposed to develop an approximate but efficient likelihood function constructed on the distance between model predictions and experimental observations. The distance-based ABC is of central importance in this framework since it acts as the connection between the Bhattacharyya distance metric and the Bayesian updating tool.

The present development is particularly motivated by the NASA UQ challenge problem [2], since it has revealed directions for improvement of current model updating technologies when they meet real-size practical problems. Herein, we demonstrate the advantages of the Bhattacharyya distance metric in association with the proposed ABC updating approach, specifically on the NASA UQ challenge. We focus on solving Sub-problem A (uncertainty characterization) out of the overall challenge problem. Sub-problem A is closely related to model updating, and its results can be significantly influenced by different UQ metrics. Three of the works previously published on solving the NASA UQ challenge problem, namely the papers by Patelli et al. [14], Ghanem et al. [15], and Safta et al. [16], are used as reference works to assess the updating results. All of these three works are also employing Bayesian updating methodologies. However, since the employed UQ metrics and the associated individual updating strategies vary, the results exhibit considerable discrepancies. This underlines the significance of using a powerful UQ metric. The proposed approach using the Euclidian and the Bhattacharyya distances in sequence as metrics in a two-step updating procedure shows clear advantages in this context.

In Section 2 we describe the theoretical and methodological basis of the Bhattacharyya distance evaluation and the Bayesian model updating. Section 3 outlines the novel developments of the distance-based ABC likelihood function, and the proposed two-step ABC updating framework. The principle and illustrative applications are detailed in Section 4, using a simple spring-mass example for illustration, and in Section 5, concentrating on to the demonstration of the performance of the framework on the highly challenging NASA UQ problem. Conclusions are drawn in Section 6.

2. Theories and methods

2.1. Formulations of the Bhattacharyya distance in UQ

In the context of stochastic model updating with UQ, the investigating system is characterized using three components: input parameters $\boldsymbol{\theta}$, output features $\mathbf{x}$, and simulator $h(\cdot)$:

$$\mathbf{x} = h(\theta) \tag{1}$$

where $\mathbf{x} = [x_1, x_2, \ldots, x_m]$; $\theta = [\theta_1, \theta_2, \ldots, \theta_n]$; m and n are respectively the number of outputs and inputs. The simulator is usually presented as either a sophisticated code package (e.g. finite element model) or a simplified function (e.g. response surface model).