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Brief paper

Stabilization of decentralized descriptor-type neutral time-delay systems by time-delay controllers[☆]

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ABSTRACT

Stabilization of non-impulsive descriptor-type linear time-invariant (LTI) neutral time-delay systems by decentralized feedback is considered. It is shown that, provided that the stability axis is to the right of the finite-spectrum abscissa of the system, there exists a stabilizing decentralized finite-dimensional LTI or a decentralized descriptor-type LTI neutral time-delay controller for such a system if and only if the system does not have any unstable decentralized fixed modes.

1. Introduction

For many large-scale systems, it may be very costly, if not impossible, to collect all the information in a centralized place, process it there, and dispatch the control commands from there. *Decentralized control* is either preferable or necessary for such systems (Jamshidi, 1997; Lunze, 1992; Šiljak, 1991). In the stabilization and mode placement of decentralized control systems, the notion of decentralized fixed modes, which was first introduced by Wang and Davison (1973), plays a central role. A *decentralized fixed mode* (DFM) is a mode of a linear time-invariant (LTI) dynamic system which cannot be moved by decentralized static output feedback. Furthermore, a complex number is said to be a μ-DFM if it is a DFM with real part greater than or equal to μ (Momeni & Aghdam, 2008a).

Many dynamic systems may involve time-delays either inherently or due to delays in communication channels, etc. (Niculescu, 2001). Such systems may be described by delay-differential or delay-differential-algebraic equations (Zhu & Petzold, 1997). When the system dynamics can be described by a delay-differential equation where all the derivatives of the state variables appear with delay-free terms, such a system is named as a *retarded time-delay system*. When all the highest derivatives appear with delay-free terms, together with some delayed terms, such a system can be named as a *non-descriptor-type neutral time-delay system*. However, when some of the highest derivatives do not appear with a delay-free term or when the system behaviour is described by delay-algebraic equations besides delay-differential equations, such a system can be named as a *descriptor-type neutral time-delay system*. Although the subject of decentralized control of finite-dimensional systems has found place in the literature for the past four decades, the consideration of the same problem for time-delay systems has been relatively new (e.g., see Bakule, 2008; Mahmoud & Almutairi, 2009; Xu & Lam, 1999). Furthermore, to the authors' best knowledge, this consideration has been restricted to non-descriptor-type (in fact, mostly to retarded-type) time-delay systems. It was established by Momeni and Aghdam (2008a) that a LTI decentralized retarded time-delay system with commensurate[1] time-delays can be μ-stabilized by LTI decentralized finite-dimensional dynamic controllers if and only if it does not have any μ-DFMs. The same result was generalized to systems with incommensurate time-delays by Momeni, Aghdam, and

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[1] The time-delays of a system are said to be *commensurate*, when each of them can be expressed as an integer multiple of a common divisor; otherwise, they are said to be *incommensurate*.

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Davison (2010). Then, it was shown by Erol and İftar (2013) that a LTI decentralized retarded time-delay system can be μ-stabilized by LTI retarded time-delay controllers if and only if it can be stabilized by decentralized finite-dimensional dynamic controllers, which extended a well-known result in the centralized case (Kamen, Khargonekar, & Tannenbaum, 1985) to the decentralized case. Many practical examples, however, exist, when a time-delay system can be best described by delay-algebraic equations coupled with delay-differential equations, rather than delay-differential equations alone (e.g., see the practical examples in Section 2.5 of Niculescu, 2001). Also, the dynamics of constrained mechanical systems, such as robotic manipulators (Lewis, Abdallah, & Dawson, 1993), which may also be subject to certain delays in their dynamics (Sheridan, 1992), can best be described in this way. With this motivation, in the present work, we extend the above mentioned results to descriptor-type neutral time-delay systems.

Throughout the paper, $\mathbb{C}$, $\mathbb{R}$, and $\mathbb{N}$, denote the sets of, respectively, complex numbers, real numbers, and non-negative integers. For $s \in \mathbb{C}$, $\mathrm{Re}(s)$ denotes the real part of s. For $k, l \in \mathbb{N}$, $\mathbb{R}^k$ and $\mathbb{R}^{k\times l}$ denote the spaces of, respectively, k-dimensional real vectors and $k \times l$-dimensional real matrices. I_k and $0_{k\times l}$ respectively denote the $k \times k$-dimensional identity and the $k \times l$-dimensional zero matrices. When the dimensions are apparent, we use I and 0 to denote respectively the identity and the zero matrices. For $\mu \in \mathbb{R}$, $\mathbb{C}^+_\mu := \{s \in \mathbb{C} \mid \mathrm{Re}(s) > \mu\}$ and $\bar{\mathbb{C}}^+_\mu := \{s \in \mathbb{C} \mid \mathrm{Re}(s) \geq \mu\}$. A transfer function matrix (TFM) $G(s)$ is said to be *real* if $G(\bar{s}) = \overline{G(s)}$ for all $s \in \mathbb{C}$, where $\bar{\cdot}$ indicates the complex-conjugate. $\mathcal{H}^\infty_{\mathbb{R},\mu}$ denotes the set of all real TFMs which are analytic and bounded on $\mathbb{C}^+_\mu$. $\det(\cdot)$, $\mathrm{rank}(\cdot)$, and $(\cdot)^T$ respectively denote the determinant, the rank, and the transpose of $(\cdot)$. $\mathrm{bdiag}(\cdots)$ denotes a block diagonal matrix with $(\cdots)$ on the main diagonal. Finally, i denotes the imaginary unit.

2. Problem statement

Consider a decentralized LTI descriptor-type neutral time-delay system $\tilde{\Sigma}$, with ν control agents, described as

$$\begin{aligned}
&\sum_{i=0}^{\sigma}\left(\tilde{E}_i\dot{\xi}(t-h_i)\right) = \sum_{i=0}^{\sigma}\left(\tilde{A}_i\xi(t-h_i) + \sum_{j=1}^{\nu}\tilde{B}_{j,i}u_j(t-h_i)\right)\\
&y_j(t) = \sum_{i=0}^{\sigma}\left(\tilde{C}_{j,i}\xi(t-h_i) + \sum_{k=1}^{\nu}\tilde{D}_{j,k,i}u_k(t-h_i)\right),\\
&\quad j = 1,\ldots,\nu,
\end{aligned} \tag{1}$$

where $\xi(t) \in \mathbb{R}^{\tilde{n}}$ is the state vector at time t, and $u_j(t) \in \mathbb{R}^{p_j}$ and $y_j(t) \in \mathbb{R}^{q_j}$ are, respectively, the input and the output vectors at time t, accessible by the j^{th} control agent ($j = 1, \ldots, \nu$). The matrices $\tilde{E}_i$, $\tilde{A}_i$, $\tilde{B}_{j,i}$, $\tilde{C}_{j,i}$, and $\tilde{D}_{j,k,i}$ ($i = 0, \ldots, \sigma, j, k = 1, \ldots, \nu$) are constant real matrices. $0 = h_0 < h_1 < \cdots < h_\sigma$ are the time-delays (for notational convenience, we use $h_0 = 0$; i.e., $i = 0$ in (1) corresponds to the delay-free part of the system), where σ is the number of distinct time-delays involved. Note that, we do not make any distinction between the commensurate and incommensurate time-delays; i.e., some of the time-delays $h_1, \ldots, h_\sigma$ may be commensurate, while others are incommensurate. Furthermore, contrary to the usual assumption, we do not require $\mathrm{rank}(\tilde{E}_0) = \tilde{n}$; i.e., we allow descriptor-type systems. However, we restrict ourselves to *non-impulsive descriptor systems*, i.e., to systems which do not produce an impulsive response to any initial conditions. Such systems can be represented as in (1), where (by an appropriate state transformation) $\tilde{E}_0$ and $\tilde{A}_0$ can be written in the form (Duan, 2010):

$$\tilde{E}_0 = \begin{bmatrix}\tilde{E}_0^{11} & 0\\ 0 & 0_{\tilde{n}_2\times\tilde{n}_2}\end{bmatrix} \quad\text{and}\quad \tilde{A}_0 = \begin{bmatrix}\tilde{A}_0^{11} & 0\\ \tilde{A}_0^{21} & \tilde{A}_0^{22}\end{bmatrix}, \tag{2}$$

where $\tilde{E}_0^{11}$ and $\tilde{A}_0^{11}$ are $\tilde{n}_1\times\tilde{n}_1$ dimensional and $\tilde{A}_0^{22}$ is $\tilde{n}_2\times\tilde{n}_2$ dimensional with $\mathrm{rank}(\tilde{E}_0^{11}) = \tilde{n}_1$ and $\mathrm{rank}(\tilde{A}_0^{22}) = \tilde{n}_2$, where $\tilde{n}_1, \tilde{n}_2 \in \mathbb{N}$ and $\tilde{n}_1 + \tilde{n}_2 = \tilde{n}$.

By defining $\zeta_i(t) := \xi(t-h_i)$, $\upsilon_{j,i}(t) := u_j(t-h_i)$, for $i = 1, \ldots, \sigma$, $j = 1, \ldots, \nu$, and

$$x(t) := \left[\xi^T(t)\ \zeta_1^T(t)\cdots\zeta_\sigma^T(t)\ \upsilon_{1,1}^T(t)\cdots\upsilon_{\nu,\sigma}^T(t)\right]^T,$$

where $x(t) \in \mathbb{R}^n$ is the new state vector at time t, we can equivalently represent the system (1) as

$$\begin{aligned}
&\sum_{i=0}^{\sigma}\left(E_i\dot{x}(t-h_i)\right) = \sum_{i=0}^{\sigma}\left(A_ix(t-h_i) + \sum_{j=1}^{\nu}B_{j,i}u_j(t-h_i)\right)\\
&y_j(t) = C_jx(t) + D_{j,k}u_j(t), \quad j = 1,\ldots,\nu,
\end{aligned} \tag{3}$$

where $E_0 = \mathrm{bdiag}\left(\tilde{E}_0, 0\right)$, $A_0 = \mathrm{bdiag}\left(\tilde{A}_0, -I\right)$, etc. We will work with the representation (3), rather than (1), since it is easier to represent the closed-loop systems under this representation. Note that, (2) implies

$$E_0 = \begin{bmatrix}E_0^{11} & 0\\ 0 & 0_{n_2\times n_2}\end{bmatrix} \quad\text{and}\quad A_0 = \begin{bmatrix}A_0^{11} & 0\\ A_0^{21} & A_0^{22}\end{bmatrix}, \tag{4}$$

where E_0^{11} and A_0^{11} are $n_1 \times n_1$ dimensional and A_0^{22} is $n_2 \times n_2$ dimensional with $\mathrm{rank}(E_0^{11}) = n_1 = \tilde{n}_1$ and $\mathrm{rank}(A_0^{22}) = n_2$, where $n_1, n_2 \in \mathbb{N}$ and $n_1 + n_2 = n$. Relating to the system described by (3), which we will denote by Σ, let us first present the following definitions.

Definition 1. For any given $\mu \in \mathbb{R}$, the set of μ-*modes* of the system Σ is defined as $\Omega_\mu(\Sigma) := \left\{s \in \bar{\mathbb{C}}^+_\mu \mid \phi_\Sigma(s) = 0\right\}$, where $\phi_\Sigma(s) := \det\left(s\bar{E}(s) - \bar{A}(s)\right)$ is the *characteristic function* of the system Σ, where $\bar{E}(s) := \sum_{i=0}^{\sigma} E_ie^{-sh_i}$ and $\bar{A}(s) := \sum_{i=0}^{\sigma} A_ie^{-sh_i}$.

Definition 2. The *finite-spectrum abscissa* of the system Σ is defined as $\mu_f(\Sigma) := \inf\{\mu_o \in \mathbb{R} \mid \text{for any } \mu > \mu_o, \Omega_\mu(\Sigma) \text{ is a finite set}\}$.

Remark 1. For a non-descriptor-type neutral time-delay system, Σ_n, which is described by (3) with $\mathrm{rank}(E_0) = n$, it can be shown that (e.g., see Michiels & Niculescu, 2007)

$$\mu_f(\Sigma_n) = \sup\left\{\mathrm{Re}(s) \mid \det\left(\sum_{i=0}^{\sigma} E_ie^{-sh_i}\right) = 0\right\}. \tag{5}$$

In particular, for a retarded time-delay system, Σ_r, which is described by (3) with $\mathrm{rank}(E_0) = n$ and $E_i = 0, i = 1, \ldots, \sigma$, $\mu_f(\Sigma_r) = -\infty$. For a so-called *lossless propagation* time-delay system (Niculescu, 2001), Σ_l, which is described by (3) with $E_i = \begin{bmatrix}E_i^{11} & 0\\ 0 & 0_{n_2\times n_2}\end{bmatrix}$, $A_i = \begin{bmatrix}A_i^{11} & A_i^{12}\\ A_i^{21} & A_i^{22}\end{bmatrix}$, $i = 0, \ldots, \sigma$, where E_i^{11}'s and A_i^{11}'s are $n_1 \times n_1$ dimensional and A_i^{22}'s are $n_2 \times n_2$ dimensional ($n_1, n_2 > 0, n_1 + n_2 = n$), where $\mathrm{rank}(E_0^{11}) = n_1$ and $\mathrm{rank}(A_0^{22}) = n_2$, it can be shown that $\mu_f(\Sigma_l) = \max\{\mu_E, \mu_A\}$, where μ_E and μ_A are defined similar to $\mu_f(\Sigma_n)$ in (5) with E_i replaced by E_i^{11} and A_i^{22} respectively, $i = 0, \ldots, \sigma$. Also note that, for any proper Σ, which satisfies $\sup_{\mathrm{Re}(s)\geq\rho}\|(s\bar{E}(s) - \bar{A}(s))^{-1}\| < \infty$, for some $\rho \in \mathbb{R}$, $\mu_f(\Sigma) < \infty$.

Definition 3. For any given $\mu \in \mathbb{R}$, the system Σ is said to be μ-*stable* if $\Omega_{\mu-\epsilon}(\Sigma) = \emptyset$ for some $\epsilon > 0$. Furthermore, a controller K is said to μ-*stabilize* the system Σ, if the closed-loop system obtained by applying the controller K to system Σ is μ-stable.