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Analytical investigation of railway overhead contact wire dynamics and comparison with experimental results

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A B S T R A C T

In this paper an analytical method for studying the free response of continuous vibrating systems with distributed and possibly non-proportional viscous damping is proposed. The most general case the method refers to is a piece-wise homogeneous Euler-Bernoulli beam, with lumped elastic and inertial elements and subjected to tensile load. The practical application of the method to a contact wire is also presented, aiming at analysing its dynamic response. Contact wires are typically used in the overhead contact line of the railway electrification system but, despite their wide diffusion, their damping properties have not been exhaustively studied. This study aims at experimentally validate the analytical method to define a reliable dynamic model of overhead contact lines.

The wire is modelled as an axially loaded homogeneous beam, with lumped elastic and inertial elements (i.e. droppers and clamps). A state-form expansion applied in conjunction with a transfer matrix technique is adopted to extract the eigenvalues and to express the eigenfunctions in analytical form. Experimental measurements have been carried out in the Dynamics & Identification Research Group (DIRG) laboratory of Politecnico di Torino considering two different damping scenarios, and the modal properties of the test bench have been extracted by using a linear subspace identification technique. The damping distribution is finally investigated starting from the experimental data, in order to seek for the most appropriate damping model.

1. Introduction

Contact wires are widely used in the overhead contact lines of railway electrification systems, and typically consist of long and flexible copper cables, with a particular cross section to allow the connection with the droppers. An extensive literature has been produced about the characterization of contact wires and overhead contact lines. Indeed, tests on the field are quite difficult to be performed and a proper model is fundamental in order to correctly simulate the dynamic evolution of the system. In [1] general indications about the modeling of an overhead contact line are presented, and a comparison between a string model and a Euler-Bernoulli beam model is carried out. The latter proved to be more effective, even because its dispersive wave characteristics better represents the behavior of the contact wire [2]. In [3] an analytical study on the effects of a moving force, representing the pantograph, has been conducted adopting a beam model. In spite of the high number of studies on the dynamics of contact wires, their damping properties have been not exhaustively investigated. Overhead contact lines are usually considered low-damped systems, and in many cases damping is suggested to be negligible at all [4].

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This is a gross approximation that can lead to unreliable results, especially when adopting a finite element model [5]. In particular, the influence of damping has been studied in [6,7], showing its considerable effects on the quality of the current collection for high speed trains. Recently, a benchmark has been proposed [8] to model the pantograph-catenary interaction. Proportional structural damping is considered in this study with coefficients $\alpha = 0.0125\ s^{-1}$, $\beta = 10^{-4}$ s obtained from measurements on the Italian high-speed catenary. In [9] the identification of damping of Norwegian overhead contact lines has been performed under different operational conditions. In [10] experimental measurements of the damping ratios have been conducted, leading to define a range from 0.01 to 0.04.

In this paper, a twofold purpose is chased: presenting an analytical method for better investigating the damping distribution and using experimental measurements to validate the predictions of the model.

The presented method can generally handle several kinds of continuous vibrating systems with either proportional and non-proportional damping. It is based on [11,12], and uses a partition of the continuous system in homogeneous substructures (or segments) in conjunction with a transfer matrix technique. In the particular case of overhead contact lines, each section corresponds to the distance between two consecutive droppers, the latter being modeled as lumped elastic elements. A key feature of overhead contact lines is the tensile force acting on both contact wire and messenger wire, thus the reference method is here extended to account for an axial load across the segments of the considered structure. The proposed approach leads to an easy implementation and presents a high computational efficiency, due to the invariance of the matrix dimensions with respect to the number of segments considered. Experimental measurements have also been performed at the DIRG laboratory of the Politecnico di Torino considering two different damping scenarios. A linear subspace identification technique [13–15] is used to extract the modal parameters from the acquired data, and a model updating process is implemented to find the best-fit between experimental results and analytical predictions. The damping distribution is finally analyzed combining information from both the experimental outcomes and the presented analytic model.

2. Modal analysis of continuous systems with viscous generalized damping and tensile force

The dynamic behavior of a continuous system with viscous generalized damping can be described, recalling [11], by the following equation of motion:

$$M\left[\frac{\partial^2}{\partial t^2}w(\mathbf{x},t)\right] + C\left[\frac{\partial}{\partial t}w(\mathbf{x},t)\right] + K[w(\mathbf{x},t)] = f(\mathbf{x},t), \quad \mathbf{x} \in \mathcal{D} \tag{1}$$

where M, C, K, are linear homogeneous differential operators and are referred to as mass, damping and stiffness operator respectively, f is the external force density, w and $\mathbf{x}$ are the displacement and the spatial coordinate in a domain of extension $\mathcal{D}$, and t is time.

The differential eigenvalue problem associated with Eq. (1) has been already solved in [11] considering a piece-wise constant Euler-Bernoulli beam with both internal and external damping distribution. In this case, the mass and the stiffness operator are:

$$M = m(x), \quad K = \frac{\partial^2}{\partial x^2}\left[k(x)\frac{\partial^2}{\partial x^2}\right] \tag{2}$$

where $m(x)$ is the mass per unit length of the beam and $k(x) = EI(x)$ is the bending stiffness, E being the Young's Modulus and I the area moment of inertia.

The damping operator can be a general external distributed viscous damping function $C = c_{ex}(x)$ or can be expressed according to the Kelvin-Voigt model [16] as:

$$C = \frac{\partial^2}{\partial x^2}\left[c_{in}(x)\frac{\partial^2}{\partial x^2}\right] \tag{3}$$

In particular, damping is said to be proportional when the damping operator can be expressed as a linear combination of the mass operator and the stiffness operator.

According to the equations above, considering both an external and an internal damping distribution, the equation of a Euler-Bernoulli beam in bending vibration under a distributed transverse force is obtained.

$$m(x)\frac{\partial^2 w}{\partial t^2} + c_{ex}(x)\frac{\partial w}{\partial t} + \frac{\partial^2}{\partial x^2}\left[c_{in}(x)\frac{\partial^2}{\partial x^2}\left(\frac{\partial w}{\partial t}\right)\right] + \frac{\partial^2}{\partial x^2}\left[k(x)\frac{\partial^2 w}{\partial x^2}\right] = f \tag{4}$$

where $f = f(x,t)$ is the transverse force and $w = w(x,t)$ is the transverse displacement. If a tensile load is applied to the beam, the equation is modified in this paper to include the axial force T (positive in tension), as follows:

$$m(x)\frac{\partial^2 w}{\partial t^2} + c_{ex}(x)\frac{\partial w}{\partial t} + \frac{\partial^2}{\partial x^2}\left[c^k_{in}(x)\frac{\partial^2}{\partial x^2}\left(\frac{\partial w}{\partial t}\right)\right] - c^T_{in}\frac{\partial^2}{\partial x^2}\left(\frac{\partial w}{\partial t}\right) + \frac{\partial^2}{\partial x^2}\left[k(x)\frac{\partial^2 w}{\partial x^2}\right] - \mathrm{T}\frac{\partial^2 w}{\partial x^2} = f(x,t) \tag{5}$$