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An explicit method for simulating non-Gaussian and non-stationary stochastic processes by Karhunen-Loève and polynomial chaos expansion

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ABSTRACT

A new method is developed for explicitly representing and synthesizing non-Gaussian and non-stationary stochastic processes that have been specified by their covariance function and marginal cumulative distribution function. The target process is firstly represented in the Karhunen-Loève (K-L) series form, the random coefficients in the K-L series is subsequently decomposed using one-dimensional polynomial chaos (PC) expansion. In this way, the target process is represented in an explicit form, which is particularly well suited for stochastic finite element analysis of structures as well as for general purpose simulation of realizations of these processes. The key feature of the proposed method is that the covariance of the resulting process automatically matches the target covariance, and one only needs to iterate the marginal distribution to match the target one. Three illustrative examples are used to demonstrate the proposed method.

1. Introduction

Problems involving random processes are commonly encountered in various fields of engineering. The modelling of parameters of input and/or system itself by means of stochastic processes increases significantly the size and complexity of the analysis. This is why the application of random field theory to engineering problems has gained considerable interest [1–4]. Except for the narrow class of problems that can be solved analytically, the solution to the variety of complex engineering problems involving uncertainty regarding mechanical properties and/or the excitations they are subjected to must be found by means of Monte Carlo simulation (MCS) [5,6]. One of the most important stages of the MCS is the generation of sample realizations of these random properties, which must accurately describe the probabilistic characteristics of the corresponding stochastic processes or fields. On the other hand, in the context of stochastic finite element (SFE) analysis of structures, the discretization of continuous stochastic processes becomes necessary [7,8]. For computational purposes, an analytical representation of stochastic processes in forms that are suitable for further mathematical manipulation is particularly preferred [9,10]. For example, in the widely used Galerkin-type method, e.g., spectral stochastic finite element method, the global stiffness matrix for uncertain stiffness has to be explicitly decomposed so that the governing algebra equation that involves the discretized stiffness matrix can be analytically solved in the subsequent Galerkin projection procedures [11,12]. With these motivations, the efficient simulation and representation of a general stochastic processes is of practical and theoretical importance.

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At present, methods for simulating Gaussian processes are quite well established owing to the fact that they are completely described by their second-order statistics [13,14]. Efforts have been also made to develop methods for simulating non-Gaussian processes since an ever-growing number of experimental measures has highlighted the non-Gaussian properties of natural phenomena. The simulation of non-Gaussian processes is a significantly more challenging and practically important problem, but its progress has been slow given the complexity of non-Gaussian stochastic processes [15–17]. The serious difficulty in dealing with non-Gaussian processes has been the complexity of their characterization. Indeed, all the joint multi-dimensional density functions are needed to completely characterize a stochastic process, and while this information is encapsulated in the second-order statistics for Gaussian processes, no such simplification exists for general non-Gaussian processes. The problem is even further complicated when the process is also non-stationary since the marginal distributions depends on time [18–22].

In the past twenty years, the simulation of non-Gaussian and non-stationary stochastic processes has spawned the development of methods rooted in two different basic simulation methods: the spectral representation (SR) method (discretization in the frequency domain) and Karhunen-Loève (K-L) expansion (discretization in the physical domain, e.g., time or space). The SR method was initially developed for simulating stationary and non-stationary Gaussian processes. Combining the SR method with Grigoriu's translation process theory enables the simulation of stochastic processes according to a prescribed power spectral density (PSD) function and marginal non-Gaussian cumulative distribution function [23–25]. The challenge with these translation-based methods is to identify an underlying Gaussian PSD that, when mapped using translation processes theory, yields a non-Gaussian PSD that matches the prescribed target [26,27]. Another important class of methods is based on the K-L expansion, which is also initially utilized for simulating stationary and non-stationary Gaussian processes [28–30]. Phoon et al. extended the K-L expansion to simulate sample functions of non-Gaussian process by iteratively updating the distribution of the underlying non-Gaussian K-L random variables [31]. This method has been further improved for simulating highly skewed non-Gaussian marginal distributions and thus offering a unified framework for the simulation of multi-dimensional homogeneous and non-homogeneous random fields [32–34]. Other techniques have also been developed for simulation of non-Gaussian processes. One important group of such techniques are those utilizing polynomial chaos (PC) decomposition such as those by Sakamoto and Ghanem [35,36]. The method represents the process as a multidimensional Hermite polynomial in a set of normalized Gaussian variables. The accuracy of this representation was extensively examined by Field and Grigoriu [37].

Based on the K-L and PC expansion, this paper presents a new method for explicitly simulating non-Gaussian and non-stationary stochastic process that has been specified by its covariance function and marginal non-Gaussian cumulative distribution function (CDF). The basic idea is to firstly represent the target process in the K-L series form, and then expand the random coefficients in K-L series using one-dimensional PC expansion. In this way, the target process is represented in an explicit form, and as a result, the commonly arised the non-Gaussian distributed global stiffness matrix can be explicitly formulated in the SFE governing equations. Therefore, the proposed method is particularly well suited for SFE analysis of structures in the calculation of response variability as well as the simulation of realizations of stochastic processes. One of the advantages of the proposed method is that, by virtue of the orthogonal property of the Hermite polynomials with respect to the Gaussian measure, the covariance of the resulting process automatically matches the target covariance, and one only needs to iterate the marginal distribution to match the target one. Thus, the proposed method offers an explicit and unified framework for simulation of non-Gaussian processes with arbitrary covariance function, which need not be stationary. Since K-L expansion is optimal among series expansion methods in the global mean square error with respect to the number of random variables in the representation, the proposed method is capable of representing a stochastic process with sufficient accuracy with as few random variables as possible, which is also quite appropriate and necessary for efficient SFE analysis and MCS of structures [10,36].

The paper is organized as follows: the present non-Gaussian K-L expansion is briefly introduced in Section 2, followed by the new method for simulating non-Gaussian and nonstationary stochastic processes. Three illustrative examples are finally given in Section 4 to demonstrate the application of the proposed method.

2. Non-Gaussian Karhunen-Loève expansion

The K-L expansion is a series expansion method for the representation of the stochastic processes. The expansion is based on a spectral decomposition of the covariance function of the process [10]. It states that a second-order process $w(x,\theta)$, which is indexed on a bounded domain $\mathscr{D}$, can be approximated by the following truncated K-L series

$$\hat{w}(x,\theta) = \bar{w}(x) + \sum_{n=1}^{M} \sqrt{\lambda_n}\xi_n(\theta) f_n(x) \tag{1}$$

where $\bar{w}(x)$ is the mean function of the process, M is the number of terms of K-L series, $\xi_n(\theta)$ is a set of uncorrelated random variables with zero mean and unit variance given by

$$\xi_n(\theta) = \frac{1}{\sqrt{\lambda_n}} \int_{\mathscr{D}} [w(x,\theta) - \bar{w}(x)] f_n(x) dx \tag{2}$$