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Non-asymptotic model quality assessment of transfer functions at multiple frequency points[☆]

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ABSTRACT

In this paper we develop methods for evaluating uncertainties in the frequency response of a dynamical system based on finitely many input–output data points. We extend the "Leave-out Sign-dominant Correlation Regions" (LSCR) algorithm to deliver confidence regions with a guaranteed probability for the frequency response at multiple frequencies, and we introduce a computationally efficient scheme that enables the confidence regions to be constructed frequency by frequency. Simulation examples illustrating the usefulness of the developed algorithm are provided.

1. Introduction

In system identification, providing a description of the uncertainties associated with the nominal system model is as important as obtaining the nominal model itself, especially for the synthesis of robust controllers. A popular technique for evaluating the model quality is based on constructing confidence regions using asymptotic system identification theory. This is a mature approach and the confidence regions can be computed relatively easily (see Ljung, 1999). However, in some cases using asymptotic theory may lead to unreliable results (see Garatti, Campi, & Bittanti, 2004) when applied to a finite number of data points.

In this paper, we consider a method for constructing confidence regions based on finitely many data points as, e.g., considered in Bayard (1993), Campi and Weyer (2005), den Dekker, Bombois, and Van den Hof (2008), Goodwin, Gevers, and Ninnes (1992) and Hjalmarsson and Ninness (2006). Unlike methods based on asymptotic theory, the developed method generates guaranteed confidence regions for a finite number of data points. The developed approach is based on the LSCR method introduced in Campi and Weyer (2005) (see also Campi, Ko, & Weyer, 2009 and Campi & Weyer, 2010), and it is extended to produce guaranteed confidence regions for the frequency response of a dynamical system. As a finite number of data points does not provide any information about the tail of the impulse response, prior information, such as exponentially decaying bounds, is introduced and incorporated in the algorithm in order to deal with tail effects. Moreover, an experimental scheme is derived that allows the confidence regions to be constructed separately frequency by frequency. This reduces the computational burden significantly.

In the next subsection we give simple preview examples that illustrate the main ideas of the proposed approach. Then, in Section 2, the procedure used in the preview examples is generalized to construct simultaneous confidence regions when the system is excited by a multi-sine input signal. In Section 3 an experimental scheme and an algorithm that allow the confidence regions to be constructed at low computational costs are introduced. Two simulation examples demonstrating the usefulness of the proposed approach are given in Section 4.

1.1. Preview examples

In this section we first introduce a simple example illustrating the main ideas of LSCR by generating a confidence interval for the

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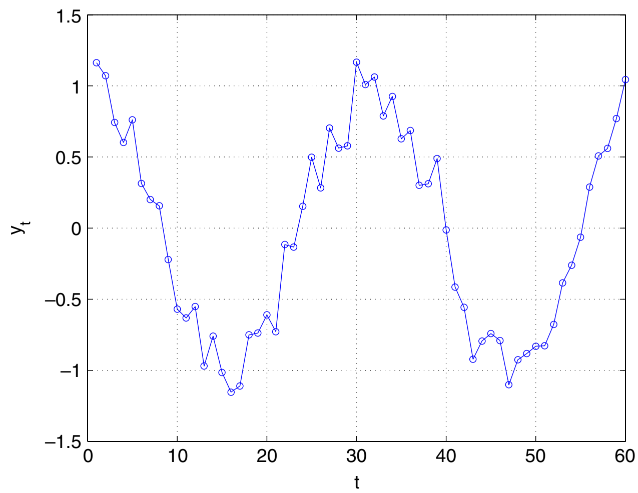

Fig. 1. Observed signal.

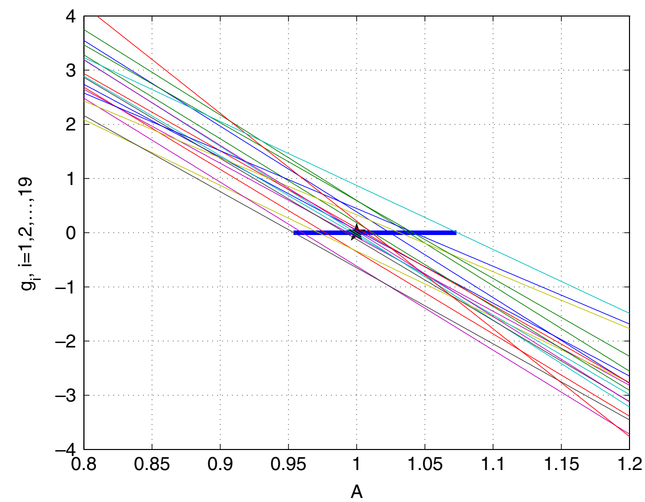

Fig. 2. $g_i(A)$ functions together with the confidence interval (thick line) and the true amplitude (★).

amplitude of a sinusoid, before moving on to the construction of a confidence set for the frequency response of a dynamical system at a given frequency. For further descriptions of the main ideas in the LSCR algorithm, the reader is referred to Campi and Weyer (2006) and Section 1.2 of Campi et al. (2009).

1.1.1. Confidence interval for the amplitude of a sinusoid

The signal of interest is a sinusoid observed in noise

$$y_t = A^0 \cos \omega t + n_t.$$

We have observations y_t, $t = 1, \dots, N = 60$. n_t is a sequence of zero mean independent random variables, symmetrically distributed about zero. The frequency $\omega = 0.2$ is known, but the amplitude A^0 is unknown. The observed signal is shown in Fig. 1. We wish to construct a confidence interval for A^0. Given the signal model

$$\hat{y}_t(A) = A \cos \omega t,$$

we compute the observation error

$$\varepsilon_t(A) = y_t - \hat{y}_t(A) = (A^0 - A)\cos \omega t + n_t,$$

and correlate it with $\cos \omega t$, which gives

$$f_t(A) = \varepsilon_t(A)\cos \omega t = (A^0 - A)\cos^2 \omega t + n_t \cos \omega t.$$

We note that $E\{\sum_{t=1}^N f_t(A)\} = 0$ for $A = A^0$, and is different from zero for $A \neq A^0$. The idea is now to use random subsamples of $f_t(A)$ to form empirical estimates of the correlation between the observation error and $\cos \omega t$. To this end we compute $M = 20$ empirical subsample estimates

$$g_i(A) = \sum_{t=1}^N h_{i,t} f_t(A) = \sum_{t=1}^N h_{i,t}\varepsilon_t(A)\cos \omega t, \quad i = 0, 1, \dots, M-1,$$

where $h_{i,t}$ are independent and identically distributed (i.i.d.) random variables taking on the values 0 and 1 with probability 1/2 each. The exception is $h_{0,t}$ which is equal to zero for all t, and hence $g_0(A) \equiv 0$. This means that $h_{i,t}$ determines whether sample t is used when $g_i(A)$ is computed.

The $M - 1$ non-zero $g_i(A)$ functions are shown in Fig. 2. Corresponding to the true amplitude A^0, $g_i(A^0)$ is a sum of zero mean random variables. It is therefore unlikely that nearly all of the $g_i(A)$ functions are positive or negative for $A = A^0$, and hence we exclude those values of A where all the $g_i(A)$ functions take on positive or negative values. Thus, the confidence interval marked with a thick line in Fig. 2 is obtained by keeping those values of A for at which at least $q = 1$ of the $g_i(A)$ functions are positive and at least $q = 1$ are negative. It is shown in Theorem 1 that the constructed confidence interval contains the true amplitude ($A^0 = 1$) with probability $1 - 2q/M = 0.9$.

Next we move onto a more realistic situation where also the phase is unknown and transient effects need to be taken into account.

1.1.2. Confidence set for frequency response

Suppose that the true continuous-time system is given by

$$y(t) = \int_0^\infty g^0(\tau)u(t-\tau)d\tau + v(t), \tag{1}$$

where $g^0(\tau)$ is the impulse response function, and $v(t)$ is additive noise. The transfer function $G^0(s)$ of the system (1) is the Laplace transform of $g^0(\tau)$ given by

$$G^0(s) = \int_0^\infty g^0(t)e^{-st}dt \tag{2}$$

and in this example it is given by

$$G^0(s) = \frac{2.5}{s + 2.5}. \tag{3}$$

This information about the true system is given for completeness of description but is unknown to the user.

The input to the system is a sinusoid

$$u(t) = \begin{cases} \cos(t), & t \geq 0 \\ 0, & t < 0. \end{cases} \tag{4}$$

The output is given by

$$\begin{aligned} y(t) &= \int_0^t g^0(\tau)\cos(t-\tau)d\tau + v(t) \\ &= \text{Re}\left\{\int_0^\infty g^0(\tau)e^{-j\tau}d\tau\, e^{jt} - \int_t^\infty g^0(\tau)e^{-j\tau}d\tau\, e^{jt}\right\} + v(t) \\ &= \text{Re}\left\{G^0(j)\cdot e^{jt} - \int_t^\infty g^0(\tau)e^{-j\tau}d\tau \cdot e^{jt}\right\} + v(t) \\ &= a^0\cos t - b^0 \sin t + \bar{y}(t) + v(t), \end{aligned}$$

where $a^0 \triangleq \text{Re}\{G^0(j\cdot 1)\}$, $b^0 \triangleq \text{Im}\{G^0(j\cdot 1)\}$ and $\bar{y}(t) \triangleq -\text{Re}\{\int_t^\infty g^0(\tau)e^{-j\tau}d\tau \cdot e^{jt}\}$ represents the transient effects due to initial conditions.