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Flexural wave band gaps in a multi-resonator elastic metamaterial plate using Kirchhoff-Love theory

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A B S T R A C T

We investigate theoretically the band structure of flexural waves propagating in an elastic metamaterial thin plate. Kirchhoff-Love thin plate theory is considered. We study the influence of periodic arrays of multiple degrees of freedom local resonators in square and triangular lattices. Plane wave expansion and extended plane wave expansion methods, also known as $\omega(\mathbf{k})$ and $\mathbf{k}(\omega)$, respectively, are used to solve the governing equation of motion for a thin plate. The locally resonant band gaps for square and triangular lattices present almost the same attenuation for all examples analysed. However, square lattice presents broader Bragg-type band gaps with higher attenuation than triangular lattice. An experimental analysis is conducted with a real elastic metamaterial thin plate with resonators in a square lattice. Modal analysis and forced response are computed by finite element method. Plane wave expansion, finite element and experimental results present good agreement.

1. Introduction

Over the last decades artificial composites known as phononic crystals (PCs) consisting of scatterer (inclusion) periodic arrays embedded in a host medium (matrix) have been well studied [1–18]. They have received renewed attention because they exhibit band gaps where there are no mechanical (elastic or acoustic) propagating waves, only evanescent waves. The physical origin of phononic and photonic band gaps can be understood at the micro-scale using the classical wave theory to describe Bragg and Mie resonances based on the scattering of mechanical and electromagnetic waves propagating within the crystal [19].

In PC investigation, band gap formation is based on the Bragg scattering mechanism, whose frequency location is governed by Bragg's law, $a = n(\lambda/2), (n \in \mathbb{N}_{>0})$, where a is the lattice parameter of the periodic system and λ is the wavelength in the host material. Bragg's law implies that it is difficult to achieve a low-frequency Bragg-type band gap for small size PCs. Difficulties to design PCs with low frequency band gaps for small sizes instigated researchers to explore other dissipative mechanisms together with the effect of periodicity. In 2000, Liu and co-workers [20] proposed a locally resonant PC, also known as elastic metamaterial (EM), containing an array of localized resonant structures. Resonance-type band gaps were obtained in a frequency range two orders of magnitude lower than that given by the Bragg's limit. Locally resonant band gaps

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arise on the vicinity of the natural frequency of the resonator while Bragg-type band gaps typically occur at wavelengths of the order of unit cell size.

EMs have been proposed for instance as acoustic barriers [21,22], vibration isolators [23–27] and noise suppression devices [28,29] with emphasis on low frequency applications. There are various types of elastic structures being investigated containing an array of resonators, such as rods [30–32], beams [33–46], plates [47–56,46] and shells [57]. These local resonators can be modelled by spring-mass resonators of single degree of freedom (S-DOF) [30,33,34,36–38,41,42,47–50,53,57,55,51,46] or multiple degrees of freedom (M-DOF) [31,32,35,39,40,43–45,52,58,59,56].

Torrent and co-workers [51] studied an EM thin plate system consisting of a honeycomb arrangement of attached spring-mass resonators. From the band structure calculation, they showed the presence of Dirac points near the K point of the Brillouin zone. The presence of edge states was studied using multiple scattering theory. Haslinger et al. [56] investigated semi-infinite periodic systems of point resonators and discussed the association with the infinite systems for square and rectangular lattices. An alternative approach using Green's functions and multipole methods to obtain dispersion diagrams for S-DOF and M-DOF structured plates was employed. Pal and Ruzzene [55] studied EMs characterized by topologically nontrivial band gaps supporting backscattering suppressed edge waves. These edge waves are topologically protected and are obtained by breaking inversion symmetry within the unit cell. They considered an EM plate containing a honeycomb arrangement of resonators. The S-DOF array of resonators generates a Dirac cone whose properties may be altered by breaking inversion symmetry through introducing alternating mass values in the unit cell.

Since periodic resonator-type structures started recently to be investigated in many engineering applications for vibration control [60,61], the main purpose of this paper is to investigate the elastic band structure, also known as dispersion diagram, of flexural waves propagating in an EM thin plate with attached multiple resonators of M-DOF in the unit cell, considering square and triangular lattices. Kirchhoff-Love plate theory [62,63] is applied for thin plate modelling. Plane wave expansion (PWE) [1,2] and extended plane wave expansion (EPWE) [64–70] methods, also known as $\omega(\mathbf{k})$ and $\mathbf{k}(\omega)$, respectively, are used to predict the complex band structure of the EM thin plate.

By using a real EM thin plate an experimental test is performed. An EM thin plate with square lattice is manufactured with a polymer (Vero White Plus) in a 3D printer with UV curing technology. Simulated results with finite element (FE) method, *i.e.* frequency response function (FRF), and with PWE method, *i.e.* band structure, are compared to the experimental data. Some different behaviours and mismatches between simulated and experimental results are found. These differences are reduced after a trial-and-error model updating by varying material property parameters (Young's modulus and mass density). PWE formulated can localize band gap position and width close to the experimental and FE results.

The paper is organized as follows. Section 2 presents PWE and EPWE approaches for an EM thin plate with periodic arrays of attached M-DOF resonators based on Kirchhoff-Love plate theory [62,63]. In the following, *i.e.* Section 3, simulated examples are carried out considering some test cases: (I) single resonator of S-DOF, (II) multiple resonators of S-DOF, (III) single resonator of M-DOF and (IV) multiple resonators of M-DOF. In Section 4 an experimental validation of EM thin plate and simulated verification using FE and PWE are performed. Conclusions are presented in Section 5. Appendices A and B present some mathematical manipulations for PWE and EPWE approaches.

2. Elastic metamaterial thin plate modelling

This section presents the formulation for an EM thin plate using PWE and EPWE methods based on Kirchhoff-Love plate theory. We consider two-dimensional periodicity, *i.e.* 2D PC, isotropic elastic plate and wave propagation in the xy plane.

PWE and EPWE are semi-analytical methods used to predict the band structure of PCs and EMs. The advantage of using EPWE over PWE is that evanescent modes are obtained naturally and they are not ignored as with PWE method. PWE assumes that Bloch wave vector, *i.e.* $\mathbf{k}$, also known as wave number, is real. In addition, EPWE method is not restricted to the first Brillouin [71] zone (FBZ) [67]. Hsue and co-workers [66] proved that the evanescent modes obtained by EPWE obey Floquet-Bloch's theorem [72,73].

Some recent studies have been developed on EM modelling using PWE and EPWE methods. Xiao and co-workers proposed PWE and EPWE methods to model a metamaterial Euler-Bernoulli beam [37] and a metamaterial Kirchhoff-Love [62,63] plate [49] with attached resonators of S-DOF in the unit cell. Moreover, Torrent and co-workers [51] used the PWE method to study an EM thin plate with attached S-DOF resonators, considering honeycomb lattice. Here, we expand the formulation of Xiao et al. [49] for an EM thin plate with attached multiple resonators of M-DOF in the unit cell, considering square and triangular lattices.

Fig. 1 sketches an infinite EM thin plate with attached single resonator of M-DOF in the unit cell, considering square (a) and triangular (b) lattices. Fig. 1 ($a - b$) also represents the first irreducible Brillouin zone (FIBZ) [71] in shaded region for square and triangular lattices, respectively. The FIBZ points in Fig. 1 ($a - b$) are $\Gamma\ (0,0), \mathrm{X}\ (\pi/a, 0)$ and $\mathrm{M}\ (\pi/a, \pi/a)$ for square lattice and $\Gamma\ (0,0), \mathrm{X}\ (4\pi/3a, 0)$ and $\mathrm{M}\ (\pi/a, \pi/\sqrt{3}a)$ for triangular lattice.

In Fig. 1 ($a - b$), there is one resonator with $\overline{N}$ DOF attached at each unit cell. Each resonator has a stiffness $k_j^{(i)}$ and a mass $m_j^{(i)}$, where $j = 1$ is the index related to the jth resonator and $i = 1, 2, \dots, \overline{N}$ is the index related to the ith DOF in the jth resonator. Fig. 2 illustrates the general case, that is N resonators of $\overline{N}$ DOF attached at the same face of each unit cell, considering