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Frequency response expansion strategy for nonlinear structures

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ABSTRACT

Finite-element (FE) models of engineering structures are generally of high order to provide detailed descriptions of the structure's static and dynamic response. However, experimental testing of such structures can only obtain spatially-incomplete sets of response data. For linear structures, the response of unmeasured degrees-of-freedom (DOFs) can be estimated using linear expanded mode shapes and experimentally-extracted linear modal parameters. Nevertheless, such techniques cannot be directly applied to nonlinear structures when the responses are distorted when driven by high levels of excitation and modal superposition is no longer valid. This paper presents a novel strategy to expand spatially-incomplete measured data to the FE-modelled DOFs that is suitable for nonlinear structures. This strategy is an extension of the current linear expansion techniques; it starts with linear experimental modal analysis and conventional linear expansion technique to estimate the unmeasured responses of the underlying linear system using low-amplitude testing data. Then the nonlinear responses, measured during high-amplitude testing, are correlated with the measured underlying linear dynamics to extract the residuals, which are subsequently used to estimate the nonlinear stiffness and damping coefficients. Finally, the unmeasured nonlinear responses are expanded using the modal properties from linear experimental analysis and estimated nonlinear coefficients. The strategy is validated using two case studies: the first is a numerical two-beam example with a localised nonlinear spring; the second is an asymmetric diesis-like structure with geometrical nonlinearities, in which data from experimental testing are used.

1. Introduction

Structures are routinely assumed to be linear during the design and test process, although nonlinear behaviours, such as resonance frequency shifts [1,2] or differences in mode shapes driven by multiple levels of inputs [3,4], are occasionally observed during, for example, ground vibration testing of an aircraft. These behaviours are accounted for by assigning a large safety factor (sometimes called model factor) to a reference linear model and thereby results in a conservative design [5]. However, due to a continuous demand for more efficient and lighter-weight structures, the actual structural behaviour can be far from linear, resulting in unreliable estimation of safety factors or even failure. For such structures, it is preferable to incorporate the nonlinear dynamics in the model to capture the critical nonlinear phenomena observed during testing and to improve the prediction fidelity of its mathematical model [6–10].

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This paper focuses on the issue of spatially-incomplete measurement during testing. A finite-element model generally contains detailed geometric features and many DOFs; however, only a subset of these DOFs are measured during a test. This is due to the limited number of measurement channels, inaccessible locations as well as difficulties in measuring rotational DOFs. For linear structures, this problem can be resolved by reducing the finite-element model to the measured region [11] or expanding the measured data – frequency response functions (FRFs) [12] or mode shapes [13,14] – to the analytically-modelled region using modal decoupling and superposition techniques. Nevertheless, these techniques are not valid for nonlinear structures. One challenge is that nonlinear structures have input-dependent FRFs, and so the nonlinear parameters should be estimated before employing an expansion technique to capture this key feature. An effective approach is to use the frequency-domain nonlinear subspace identification procedure [15,16], where the nonlinear forces are regarded as feedback forces to the underlying linear system; it requires direct measurements at all locations where nonlinearities exist. Achieving this requirement is not always possible in practice, especially when the nonlinearities are distributed. An alternative approach is to extract modal properties using a dedicated nonlinear phase resonance testing technique and then synthesise the nonlinear frequency responses at measured DOFs using the nonlinear modes, as demonstrated for structures with isolated resonances by Peter et al. [17]. For a nonlinear structure with closely-spaced underlying linear modes, the nonlinear responses are complex – for example, the deflection shapes may alter significantly during testing [18] – which makes it very difficult to isolate one mode and perform the phase resonance testing. In this respect, this paper proposes a response expansion strategy to expand the measured nonlinear responses to the entire FE-modelled region. It utilises low-amplitude test data to update the FE model of the underlying linear system and perform a linear expansion using mature techniques. High-amplitude data from standard stepped-sine or swept-sine testing techniques are then used to estimate the nonlinear parameters and perform the expansion. The strategy is first demonstrated with a numerical model of two beams connected with a discrete nonlinear spring. It is then applied to the experimentally-measured data of an asymmetric, diesis-like structure with geometrical nonlinearities.

The rest of the paper is structured as follows: in Section 2, we derive the expansion strategy; Section 3 describes a numerical example of a two-beam model with a discrete nonlinear spring; the experimental testing of an asymmetric diesis-like structure with geometrical nonlinearities is then discussed in Section 4. Finally, conclusions are drawn in Section 5.

2. Theoretical background

2.1. General framework and dynamic equations

A typical measurement of a nonlinear structure can be divided into two sets of testing: low-amplitude and high-amplitude tests [19,20]. It is assumed that homogeneous FRFs, i.e., FRFs that are independent of the input spectrum [21], are obtained using low-amplitude random or sinusoidal testing and thereby the measured responses can be expanded to unmeasured regions using mature linear expansion theory [12–14]. A finite-element model of the underlying linear part of the structure is assumed to be available in the proposed nonlinear expansion technique. High-amplitude tests, e.g., stepped-sine or slowly-swept-sine excitations, drive the structure to its operational energy level where significant nonlinear distortions may be observed in the responses. This testing procedure for a nonlinear structure is appealing because the data can be obtained using conventional testing equipment in industry. This paper aims to extend the linear expansion technique to incorporate high-amplitude test data in which nonlinear behaviour is observed. Here, the measured responses are assumed to be periodic and the primary harmonic term is considered to be the dominant component, as is observed in many engineering structures [1–3,10,19,20]. Additional resonances such as the sub- and/or super-harmonic resonances are assumed to be negligible. More complicated nonlinear responses such as quasi-periodic or chaotic motions are not within the scope of the expansion technique discussed here.

As shown in Fig. 1, the response of a nonlinear structure during high-amplitude testing can be divided into *linear frequency band* (frequency bandwidth where FRFs are approximately homogeneous using multiple input forcing levels) and *nonlinear frequency band* (frequency bandwidth where homogeneous FRFs are observed using low-amplitude testing and distortions such as amplitude-dependent FRFs or jumps occur using high-amplitude testing). In the linear frequency band, the modal parameters of the nonlinear structure remain unchanged in the high-amplitude testing, i.e. the modes are not noticeably affected by nonlinearities, thus the mature linear expansion technique [12] can be applied for the data in this frequency range. On the other hand, the responses in the nonlinear frequency band require a nonlinear expansion technique, which is the main focus of this paper. The expansion techniques used in the proposed expansion strategy for a nonlinear structure are summarised in Table 1.

The proposed nonlinear expansion technique makes use of the following assumptions: 1) it is performed after the linear expansion using data from low-amplitude testing; 2) it can utilise the linear modal parameters and linear expanded data, i.e., the underlying linear responses of the unmeasured DOFs are obtained prior to the nonlinear expansion; 3) the modes outside the nonlinear frequency band are approximately linear and can be pre-identified using data from the low-amplitude testing. With these assumptions, the nonlinear expansion strategy is shown in Fig. 2.

In order to describe the expansion technique, we consider the governing equation of an N-DOF nonlinear structure

$$\mathbf{M}\ddot{\boldsymbol{x}} + \mathbf{C}\dot{\boldsymbol{x}} + \mathbf{K}\boldsymbol{x} + \boldsymbol{f}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \boldsymbol{p}(t), \tag{1}$$