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Identification of linear continuous-time systems under irregular and random output sampling[☆]

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ABSTRACT

This paper considers the problem of identifiability and parameter estimation of single-input–single-output, linear, time-invariant, stable, continuous-time systems under irregular and random sampling schemes. Conditions for system identifiability are established under inputs of exponential polynomial types and a tight bound on sampling density. Identification algorithms of Gauss–Newton iterative types are developed to generate convergent estimates. When the sampled output is corrupted by observation noises, input design, sampling times, and convergent algorithms are intertwined. Persistent excitation (PE) conditions for strongly convergent algorithms are derived. Unlike the traditional identification, the PE conditions under irregular and random sampling involve both sampling times and input values. Under the given PE conditions, iterative and recursive algorithms are developed to estimate the original continuous-time system parameters. The corresponding convergence results are obtained. Several simulation examples are provided to verify the theoretical results.

1. Introduction

System identification for continuous-time systems via sampling is a classical field (Åström & Wittenmark, 1997; Chen & Francis, 1995; Ding, Qiu, & Chen, 2009; Phillips & Nagle, 2007). It is well understood that to identify a time-invariant continuous-time system, one may derive its time-invariant discrete-time sampled system with periodic sampling and the zero-order hold; and hence identification of the original continuous-time system is converted to that of its sampled system (Åström & Wittenmark, 1997; Garnier & Wang, 2008; Ljung, 1999; Marelli & Fu, 2010). A sufficient condition to guarantee the one-to-one mapping from the coefficients of the sampled system to the original system is that the sampling period is less than an upper bound related to the imaginary parts of the poles (Ding et al., 2009). This equivalence implies that the existing algorithms for discrete-time systems suffice for identification of the original continuous-time system. Furthermore, it was shown in Ding et al. (2009) that multi-rate sampling schemes can be used to create such a one-to-one mapping when the sampling rate is slower than this bound. Under such a multi-rate sampling system, the sampled system of a linear time-invariant system remains linear and time invariant with a higher order.

In practical systems, especially networked systems, periodic sampling is no longer valid. Examples are abundant, such as communication channels with packet loss and unpredictable round-trip times. Irregular sampling time sequences may be generated passively due to event-triggered sampling (Åström & Bernhardsson, 1999), low-resolution signal quantization (Wang, Yin, Zhang, & Zhao, 2010), activities by input control or threshold adaptation

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under binary-valued sensors (Wang, Li, Yin, Guo, & Xu, 2011), or PWM (Pulse Width Modulation)-based sampling (Wang, Feng, & Yin, 2013). Under irregular or random sampling, the sampled system of a linear time-invariant system becomes time-varying, for which system conversion is complicated and computational complexity is much higher. When sampling is slower, irregular, or random, system identification formulation, identifiability, algorithms, accuracy, and convergence will be fundamentally impacted. This paper will explore related issues in this paradigm.

In Johansson (2010), the original differential equation is first converted to an algebra equation with respect to time by using filtered input and output signals. Then the parameters of the algebra equation are estimated at the irregular sampling points, and the original system parameters are recovered by a one-to-one mapping from the estimated parameters. One possible methodology is to identify the original system parameters without conversion to its sampled system (Gillberg & Ljung, 2010; Larsson & Söderström, 2002; Marelli & Fu, 2010; Vajda, Valko, & Godfrey, 1987). The parameters are directly identified by using a continuous-time frequency domain identification method in Gillberg and Ljung (2010). Similar to its discrete-time counterpart, the continuous-time system can also be expressed by a linear regression equation in a differential operator form (Larsson & Söderström, 2002), in which the regressor involves input and output signals and their derivatives. Since the derivatives are unavailable under sampled data, higher-order derivatives of the input and output signals are approximated by their related differences (Larsson & Söderström, 2002), introducing errors as a consequence. The resulting discrete-time system is then identified by batch or recursive algorithms (Ljung, 1999; Ljung & Vicino, 2005). To reduce approximation errors, fast sampling is required. An instrumental variable approach is used to enhance estimation accuracy for continuous-time autoregressive processes in Mossberg (2008), which demonstrates improved computational efficiency in comparison to the least squares approach (Larsson, Mossberg, & Soderstrom, 2007).

Synchronization between the input sampling and output sampling is also a significant factor. Typical schemes for the indirect method assume that the input and output are sampled at the same sampling points (Larsson & Söderström, 2002; Yuz, Alfaro, Agüero, & Goodwin, 2011). The estimation method in Yuz et al. (2011) represents the original continuous-time state space model by an incremental approximation under nonuniform but fast sampling and employs the maximum likelihood approach. Zhu, Telkamp, Wang, and Fu (2009) propose a two-time scale sampling scheme: fast uniform input sampling, but slow and irregular output sampling, with assumption that the output sampling time is a multiple of the input sampling time. Under an output error structure, the system parameters are then estimated by minimizing a suitable loss function. In contrast, in Gillberg and Ljung (2010) the input is a uniformly spaced piece-wise constant function (zero-order hold), while the output is sampled irregularly. The main technique is to use B-spline approximation to achieve uniformly distributed knots from the non-uniformly sampled output. The method in Gillberg and Ljung (2010) is restricted to the noise-free sampled output and its estimation accuracy enhancement requires fast sampling. Despite extensive research effort in this area, some fundamental questions remain un-answered: (1) How fast and under what types of sampling schemes, is the continuous-time system identifiable? (2) What types of inputs will imply system identifiability? (3) What modifications must be made to identification algorithms? (4) To achieve convergence, how should the input be designed?

This paper investigates these questions from a new angle. Instead of focusing on parameter mappings between the continuous-time system and its sampled system, we view irregularly or randomly sampled values as the available information set and study identifiability, identification algorithms, and input design directly on the original parameters. The main contributions of this paper are in the following aspects. (1) We show that under any inputs of exponential polynomial types, the continuous-time system is identifiable if the sampling points are sufficiently dense in a given time interval. The bound on the density of the sampling points is tight, revealing an interesting connection, in terms of identification information complexity, to Shannon's sampling theorem for signal reconstruction (Proakis & Manolakis, 2007) and our recent results on state estimation (Wang et al., 2011). Note that the input used in this paper is continuous, while the existing literature (Ding et al., 2009; Gillberg & Ljung, 2010) commonly uses piece-wise constant inputs by zero-order hold. (2) Robustness of system identifiability under a given sampling density is established. (3) Under noise-free observations, a convergent iterative algorithm is introduced, which is valid for any input signals satisfying certain gradient conditions. (4) Under noisy observations, suitable identification algorithms are proposed, which are shown to converge strongly and carry properties of the CLT (central limit theorem) types if certain ergodicity conditions are satisfied. (5) Persistent excitation (PE) conditions are derived that ensure convergence of the developed algorithms. Departing from the traditional PE conditions that rely only on input values, it is shown that under irregular or random sampling, both sampling time sequences and the input values impact on convergence. These results provide guidance for input design in identification experiments. (6) Consistency of the algorithms is proved without requiring fast sampling.

The rest of the paper is arranged as follows. The system setting and several key properties are presented in Section 2. System identifiability is investigated in Section 3. The parameter estimation algorithms and their convergence properties under noise-free observation are discussed in Section 4. When observations are noise corrupted, identification algorithms are significantly different from noise-free cases. Two kinds of estimation algorithms (iterative algorithms and recursive algorithms) are introduced and their convergence conditions are established in Section 5. Section 6 is focused on input design problems. The related persistent excitation conditions are obtained. In Section 7 some numerical examples are given to verify the effectiveness of the proposed algorithms of this paper. Section 8 concludes the paper with some further remarks. The main proofs of the assertions in the paper are placed in the Appendix.

2. Preliminaries

This section describes the system setting and establishes several important properties to be used in the subsequent sections.

2.1. Systems

We are concerned with identification of a single-input–single-output, linear, time-invariant, stable, finite dimensional system in the continuous-time domain, represented by a strictly proper transfer function

$$G(s) = \frac{b_1 s^{n-1} + \cdots + b_{n-1}s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n} \triangleq \frac{b(s)}{a(s)}, \tag{1}$$

where $a(s)$ is stable, i.e., all the roots of $a(s)$ lie on the open left-half complex plane; $a(s)$ and $b(s)$ are coprime, i.e., they do not have common roots. The impulse response of $G(s)$ is denoted by $g(t) = \mathcal{L}^{-1}(G(s))$, where $\mathcal{L}^{-1}$ is the inverse Laplace transform. Let the system parameters be expressed as $\theta = [a_1, \ldots, a_n, b_1, \ldots, b_n]'$. We use $G(s, \theta)$ and $g(t, \theta)$ to indicate their dependence on the parameters. $\mathbb{R}$ and $\mathbb{C}$ are the fields of real and complex numbers, respectively.