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Transmission-error frequency-domain-behavior of failing gears

William D. Mark [a,*], Aaron C. Isaacson [b], Matthew E. Wagner [b]

[a] *Applied Research Laboratory and Graduate Program in Acoustics, The Pennsylvania State University, University Park, PA 16802 USA*
[b] *Applied Research Laboratory, The Pennsylvania State University, University Park, PA 16802 USA*

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ABSTRACT

Failing gear teeth, either by working-surface-damage (pitting, spalling, scuffing, etc.) or by bending fatigue, causes tooth-to-tooth variations in the loaded tooth working surfaces. Such variations cause changes (generally increases) in the non-tooth-meshing rotational-harmonic amplitudes of the transmission-error contribution from the affected gear. Simple models of missing working-surface material caused by damage are used to show where transmission-error rotational-harmonic spectrum changes will take place. Bending fatigue damage is shown to initially cause maximum changes in rotational-harmonic amplitudes well below the tooth-meshing fundamental harmonic, whereas small pits are shown to cause changes in higher-frequency rotational-harmonic amplitudes. Good agreement is shown between an experimentally obtained rotational-harmonic spectrum caused by tooth-surface damage and that predicted from damage measured on the failing teeth. Substantial increases in high-frequency rotational-harmonic amplitudes are shown to be expected from gear teeth undergoing significant plastic deformation in late stages of bending-fatigue failure. Accurate assessment of damage contributions using before-damage non-negligible rotational-harmonic amplitudes (sideband harmonics, etc.) are shown to suggest use of complex rotational-harmonic amplitudes.

1. Introduction

Failing gear teeth exhibit removal of material from otherwise conjugate (involute) tooth-working-surfaces. In cases of surface damage (pitting, spalling, scuffing, etc.), direct removal of material is exhibited. In cases of tooth bending fatigue, failing teeth exhibit plastic deformation [1] which is actual removal of material, and apparent removal of material caused by reduced tooth stiffness resulting from tooth-root cracks. All such real or apparent missing tooth-working-surface material provides transmission-error contributions from meshing gear pairs.

The static transmission error (STE) is the principal vibration excitation arising from meshing gear pairs [2–9]. For fixed-axis gear pairs, the transmission path between the meshing teeth location and remote hard-mounted transducer output normally is well-modeled as a linear time-invariant system. In such situations, transducer outputs can be utilized to detect and assess the progression of gear damage contributions exhibited in the transmission error.

Many useful characteristics of gear damage show up in the frequency domain. Consider Fourier series representation of the static transmission error of a gear operating under constant loading and constant speed. The rotation period of the gear is the period of the *fundamental rotational harmonic* $n = 1$ of the STE of the gear. If the gear has N teeth, rotational-harmonic $n =$

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* Corresponding author.
E-mail addresses: wdm6@psu.edu (W.D. Mark), aci101@psu.edu (A.C. Isaacson), mew211@psu.edu (M.E. Wagner).

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Nomenclature

English

$B_{k\ell}(n)$	DFT of peak damage amplitude $c_{j,k\ell}$, Eqs. (3) and (12)
$c_{j,k\ell}$	peak value of missing material (linear dimension), Eq. (2)
D	radial span of tooth contact zone (Fig. 1)
D′	radial damage span (Fig. 1)
F	axial span of tooth contact zone (Fig. 1)
F′	axial damage span (Fig. 1)
H(n)	complex frequency response function of transmission path between teeth meshing location and transducer output, Eq. (13)
j	tooth number, j = 0, 1, 2, 3,. . ., N−1
$j_0(x)$	(sin x)/x, spherical Bessel function of order zero
n	rotational harmonic number of STE, Eq. (5)
N	number of teeth on gear of interest
Q_a	axial contact ratio of tooth contact zone
Q_t	transverse contact ratio of tooth contact zone
y	axial tooth coordinate
y_{oj}	axial center of non-centered damage
z	radial tooth coordinate
z_{oj}	radial center of non-centered damage

Greek

$\alpha_y(n)$	complex Fourier series amplitude of transducer output, Eq. (13)
$\alpha_\zeta(n)$	complex Fourier series amplitude of STE damage, Eqs. (4) and (5)
$\eta_{Cj}(y,z)$	damage-caused missing working-surface material, Eqs. (1) and (2)
$\hat{\phi}_{k\ell}\left(\frac{n}{N}\right)$	mesh-attenuation function of damage component, $k\ell$, Eq. (6)
$\psi_{yk}(y)$	axial missing material shape, $\psi_{yk}(0) = 1$, Eq. (1)
$\hat{\psi}'_{yk}\left(Q_a \frac{F'}{F}\frac{n}{N}\right)$	normalized Fourier transform of axial missing material shape [25, Eq. (40a)]
$\psi_{z\ell}(z)$	radial missing material shape, $\psi_{z\ell}(0) = 1$, Eq. (1)
$\hat{\psi}'_{z\ell}\left(Q_t \frac{D'}{D}\frac{n}{N}\right)$	normalized Fourier transform of radial missing material shape [25, Eq. (40b)]

Subscripts

a	after damage designation, Eq. (30a)
b	before damage designation, Eq. (28)
C	Cartesian coordinate designation of tooth working-surface dimensions, Eq. (1)
j	tooth number, j = 0, 1, 2, 3,. . ., N−1, Eq. (1)
k	axial damage designation, Eq. (1)
ℓ	radial damage designation, Eq. (1)
y	axial working-surface coordinate, Eq. (1)
z	radial working-surface coordinate, Eq. (1)
ζ	static transmission error, Eq. (4)

N is the *tooth-meshing fundamental harmonic.* First consider a perfect gear with identical equispaced elastically deformed teeth. The only temporal variation in the STE contribution from such a perfect gear is the variation associated with the elastically deformed perfect teeth, that is, the tooth-meshing harmonics, n = N, 2 N, 3 N, •••; n = pN, p = integer [10, pp. 112, 113]. But for present-day high-quality gears, the tooth-to-tooth working-surface variations of undamaged gears typically are very small, and as a consequence, their STE non-tooth-meshing rotational-harmonic contributions also are very small, with the possible exception of so-called "sideband" rotational-harmonic contributions located about the STE tooth-meshing-harmonics.

In contrast to undamaged gears, damage on one or a few tooth-working-surfaces will cause significant contributions to the STE non-tooth-meshing rotational-harmonics of a gear, with only small or negligible contributions to the STE tooth-meshing-harmonics. Moreover, as shown in the following pages, the *scale* of the tooth-working-surface damage, e.g. small pits versus tooth-bending damage, determines the rotational-harmonic region of STE maximum rotational-harmonic amplitudes caused by the damage.

In order to successfully detect and monitor damage progression on a gear of interest, it therefore is necessary to isolate the STE non-tooth-meshing rotational-harmonic contributions of that gear from the STE contributions of the mating gear to