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Characterization of elastic and electromechanical nonlinearities in piezoceramic plate actuators from vibrations of a piezoelectric-beam

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ABSTRACT

Piezoceramic actuators have been observed to exhibit nonlinear behaviour even when subjected to weak electric fields. The experimental observations necessitate the need for a nonlinear constitutive equation, to aptly describe the behaviour of the piezoelectric actuators. The nonlinear response could have been effected due to a nonlinear behaviour in the mechanical domain (nonlinear elastic behaviour), or due to a nonlinear electromechanical coupling, or due to a combination of both the nonlinear elastic behaviour and nonlinear electromechanical coupling. To locate the source of nonlinearity and to identify the nature of the nonlinearity, a two-step experimental procedure is proposed in this article, wherein experiments were conducted on a cantilever duralumin beam with a pair of surface bonded PZT-5H piezoelectric patches. The source of the nonlinearity in the piezoelectric actuators (as to whether it is from the mechanical domain, or in the electromechanical coupling) can be directly identified from the experimental results obtained from this procedure. The nature of the contribution of each domain, to the overall nonlinear behaviour, can be deduced from the backbone curves obtained from the piezoelectric-beam vibrations. The first-step of the experimental procedure was aimed at investigating only the mechanical domain of the piezoelectric actuator. This was achieved by short circuiting the PZT-5H patches and obtaining the displacement frequency response curves of the piezoelectric-beam by base excitation. The second-step of experiments were aimed to investigate the presence of nonlinearity in the electromechanical coupling. Here, the piezoelectric-beam was excited by the PZT-5H actuator to get a similar set of displacement frequency response curves for the first, second and third modes. The nonlinear constitutive equation was established from the profile of the backbone curves of the displacement frequency response plots obtained from the experiments. It was observed that, apart from the linear terms, both quadratic and cubic strain terms were required to represent the stress in the piezoelectric material. The electromechanical coupling too required a nonlinear representation whereby, aside from the linear relation the stress was equal to the product of cubic strain and the electric field.

1. Introduction

In most applications, linear constitutive relations [1,2] are used to describe the nature of piezoelectric actuators at weak electric field excitations. The use of linear constitutive equations obviously imply a linear behaviour of the piezoelectric

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material, which is contrary to the observations made by researchers over the past few years. As one of the first reported cases, Crawley and de Luis [3] revealed a nonlinear relation between the excitation voltage and the resulting tip amplitude of an aluminum beam with two surface bonded piezoelectric actuators. The nonlinear behaviour was attributed to the increase of structural damping with the increase of tip amplitude of the piezoelectric-beam, and not on the nonlinear nature of the piezoelectric actuators. But, later experiments conducted by Crawley and Anderson [4], and Crawley and Lazarus [5] showed the electric field-strain relation to be nonlinear, and they further determined the piezoelectric constant (d_{31}) to be dependent on the induced strain (which, in the linear constitutive equations is a constant). A nonlinear constitutive equation for the PZT material was derived by Guyomar et al. [6] based on the nonlinear phenomena observed in the Langevin transducers. They derived the equation for stress T (as a function of strain S and Electric field E) by the Taylor series at points $S = 0$ and $E = 0$, and neglected the nonlinear terms of electric filed, as the applied electric fields were low. The final stress expression used in their simulations was

$$T = cS - eE + \frac{\alpha S^2}{2} - \gamma SE.$$

This nonlinear constitutive equation was supported by numerical simulations, with the numerical results matching the patterns observed in their experimental data. In a later study, Guyomar et al. [7] reported a nonlinear experimental plot of voltage versus displacement of a piezoelectric-beam. In a similar manner, experiments conducted by von Wagner and Hagedorn [8] on a piezoelectric-beam, at the first natural frequency, also showed a nonlinear relationship between the actuation voltage and the resulting displacement. The excitation voltage in their experiments corresponds to weak electric fields, and the observed plot had a profile similar to the one reported by Crawley and de Luis [3]. They then derived the nonlinear constitutive equations of the piezoelectric material based on the experimental evidence that the Young's Modulus and the piezoelectric constant d_{31} were dependent on the strain in the piezoelectric material. The so obtained nonlinear equation for the stress in the piezoelectric actuator was,

$$T_{xx} = E_c^{(0)} S_{xx} + E_c^{(1)} S_{xx}^2 + E_c^{(2)} S_{xx}^3 - \gamma_0 E_z - \gamma_1 S_{xx} E_z - \gamma_2 S_{xx}^2 E_z.$$

They estimated the nonlinear coefficients from the experimental data and also plotted the backbone curve of the second mode (which displayed a softening behaviour). Utz von Wagner [9] also studied the nonlinearity in longitudinal vibrations of piezoelectric rods, and unlike the earlier study [8] the interest in this case was in the "33" coupling. Further work on nonlinearity in the "33" piezoelectric coupling was done by Parashar and von Wagner [10].

In most of the studies mentioned above, a plot of displacement (or strain) versus the applied voltage is used to confirm the existence of nonlinearity in the system. The same plot is subsequently used to construct the nonlinear constitutive equation for the piezoelectric actuators. A plot of the tip displacement versus the excitation voltage of a piezoelectric-beam (used in the experiments described in this article) is shown in Fig. 1. This plot is constructed from the beam's response corresponding to sinusoidal voltage excitations (at any resonance frequency, preferably the fundamental resonance) at various amplitudes. Though this plot reveals the presence of nonlinearity, an accurate quantification of the nonlinearity cannot be made from this plot due to the following reasons:

(i) The plot reveals the nonlinear relation between the applied voltage and the resulting displacement, in other words between the electric field and the resulting strain in the piezoelectric actuator, but the type of relation is not evident from the graph. The nonlinear terms relating the two quantities has to be determined from a list of possibilities by a best fit method using the experimental data. Amongst the list of candidates for describing the nonlinear phenomenon, more than one possible set of relation can describe the observed phenomenon (Fig. 1).

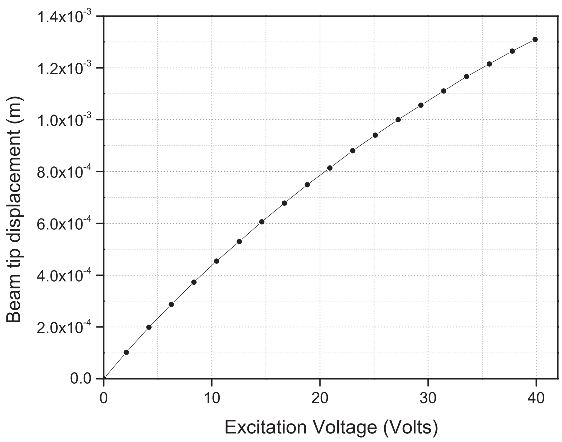

Fig. 1. Tip displacement of the piezoelectric-beam, used in the experiments presented in this article, versus the actuation voltage at a frequency of 20.225 Hz.