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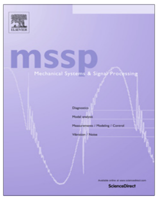

A non-probabilistic time-variant reliable control method for structural vibration suppression problems with interval uncertainties

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ABSTRACT

Active control techniques are necessary to structural vibration problems and thus studies of controller design are particularly important in mechanical dynamic engineering. Because parametric deviations due to inherent nature or external excitation are inevitable and can severely influence the final performance of real control systems, optimal active control considering uncertainty is gradually becoming a major concern in modern theory fields. In this paper, a novel non-probabilistic time-variant reliability-based optimization (NTRBO) strategy is presented for closed-loop controller design of vibration reduction issues. First, boundary rules and auto-correlation characteristics of controlled responses are confirmed based on the state-space transformation and the interval process approach. Then, enlightened by models of the first passage and the safety factor (SF), a new definition of the time-variant reliability measurement is provided. As keys to construct the optimal controller, weighing matrices in the Riccati equation are finally determined by solving the developed NTRBO model. The validity and the feasibility of the proposed methodology are demonstrated by several example applications, and the results reveal that uncertainty factors in optimal active control can be addressed from a new time-variant reliability perspective.

1. Introduction

Vibration issues are commonly inevitable in structural analysis and design, while undesired vibrations can cause different types of problems, such as fatigue damage, high-level noise, and resonance occurrences. Thereby, it is of great significance to take possible measures to achieve vibration damping and noise reduction in practical applications [1–3]. Currently, two main classes of vibration control strategies, named as passive control techniques and active control techniques have been developed [4,5]. Compared with passive means, greater attention has focused on vibration active control approaches due to the advantages of economy and flexibility in fields of dynamic mechanical engineering [6–8].

The controller, as the 'brain' of the active control system, plays an important role in vibration rejecting effects [9]. In recent years, a number of research works have examined control strategies used to design active controllers, such as pole

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placement technologies [10,11], optimal control theories [12,13], adaptive control policies [14,15], etc. Among them, the optimal control scheme, which designs the controller by solving the optimization model of weighting functions between mechanical performance and the control effort, is most popular [16]. ElBeheiry and Karnopp [17] investigated five different types of suspension systems based on the optimal control theory and developed a new linear quadratic Gaussian (LQG) algorithm for controller selection. Nagaya et al. [18] presented an optimization method for vibration absorber determination in which structural stiffness can be controlled by the microcomputer under the auto-tuning algorithm to create an anti-resonance state. Zhang [19] accomplished optimal vibration control directly utilizing the second-order dynamic equation and the feedback gain matrix was embodied by serial linear forms to avoid difficulties in solving the traditional non-linear Riccati equation. Omidi et al. [20] proposed a modified positive velocity feedback (MPVF)-based control approach that integrates the M-norm optimization model to improve the multi-mode suppression performance of the vibration controller. Maraniello and Palacios [21] applied the single shooting method in the optimal vibration control of flexible beam structures; the discussed gradient-based algorithm could overcome the disadvantages of traditional sequential design approaches. More introductions of optimal active control can also be found in the literature [22–26].

As the above literature reveals, although the theoretical development of optimal active control is quite mature in the field of structural vibration suppression problems, most control schemes are under deterministic assumptions, whereas uncertain factors originating from material dispersion, loading fluctuation, measuring deviation, etc., are inevitable in practice [27–29]. These issues must be reasonably dealt with as part of controller design instead of adopting compensation measures in a later period [30]. Existing robust control principles such as H_∞-based models generally adopt an integrated estimation of uncertainty and aim to minimize the sensitivity features of the closed-loop system to parametrical changes [31,32]. That is, the controller obtained by the robust control is always conservative, which implies an overestimation of accumulative amplification from input uncertainties to dynamic responses (contrary to the ‘optimal control’ idea) [33].

To overcome the shortcomings of conventional robust control, more refined strategies in uncertainty-oriented controller design should be presented [34,35]. Reliability-based optimization (RBO) techniques, particularly non-probabilistic reliability-based optimization (NRBO) techniques via limited uncertainty information, can precisely quantify the effect of complex uncertain factors on structural safety and further guide intelligent design. Consequently, these techniques have gradually become major topics and useful strategies in both academic and engineering circles for mechanical design tasks [27,36]. In the 1990s, Ben-Haim and Elishakoff [37,38] first gave definitions of non-probabilistic safety measures based on set-theoretical convex methods and applied them to structural shape and size optimization models. Qiu et al. [39,40] and Jiang et al. [41,42] revisited the concept of non-probabilistic reliability in a critical light and expanded the NRBO in aeronautical and vehicle engineering. Kang and Luo [43] proposed a non-probabilistic reliability-based topology optimization (NRBTO) method for the design of continuum structures undergoing large deformations. In this method, uncertain parameters are enveloped by the multi-ellipsoid convex model. Additional studies corresponding to multidisciplinary optimization and dynamic reliability were also suggested [44–47]. Consequently, we believe that non-probabilistic reliability notation and optimal controller design could be a feasible direction of development in the vibration active control field.

However, research results related to reliable controller design remain rare, and several issues, such as the accuracy of uncertainty quantification (UQ) and the validity of dynamic reliability evaluation, remain unsolved. Accordingly, this study aims at investigating a new non-probabilistic time-variant reliability based optimization (NTRBO) methodology for vibration active control problems with unknown-but-bounded (UBB) uncertainties. The remainder of this paper is organized as follows. First, the state-space representation model based on the linear quadratic regulator (LQR) control scheme is reviewed and the deterministic optimization formulation of controller design is established in section 2. To perform the time-varying UQ analysis, quantitative models of static intervals and dynamic interval processes are introduced in Section 3.1; we then employ the second-order Taylor extension approach to accurately determine boundary rules of controlled responses and embody time-dependency features by covariance and auto-correlation solutions in Section 3.2. Furthermore, inspired by the first passage and safety factor ideas, Section 4.1 defines a new non-probabilistic time-variant reliability index in a piecewise format; then, a novel NTRBO strategy and iteration algorithms are discussed in Section 4.2 to obtain an optimal reliable controller under a balance of systematic stability and economic consumption. Section 5 provides two numerical examples to verify the effectiveness of the developed method, followed by some conclusions.

2. General problem statements

2.1. Vibration response prediction under a closed-loop control scheme

Consider the dynamic equation of one n degree-of-freedom (DOF) structure under active control as

$$\begin{cases} \mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{P}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{L_e}\mathbf{f}(t) + \mathbf{L_c}\mathbf{u}(t) \\ \dot{\mathbf{x}}(t_0) = \dot{\mathbf{x}}_0, \mathbf{x}(t_0) = \mathbf{x}_0 \end{cases}, \quad t_0 \leqslant t \leqslant t_f \tag{1}$$

where $\mathbf{M}$, $\mathbf{P}$ and $\mathbf{K}$ are respectively the mass, damping and stiffness matrices; $\mathbf{x}(t)$, $\dot{\mathbf{x}}(t)$ and $\ddot{\mathbf{x}}(t)$ represent the response vectors of displacement, velocity and acceleration; $\mathbf{L_e} \in \mathbb{R}^{n\times r}$ and $\mathbf{L_c} \in \mathbb{R}^{n\times s}$ assign the position and amplitude information of the external excitation $\mathbf{f}(t) \in \mathbb{R}^{r\times 1}$ and the control force $\mathbf{u}(t) \in \mathbb{R}^{s\times 1}$; and the initial conditions $\mathbf{x}(t_0)$ and $\dot{\mathbf{x}}(t_0)$ should be given in advance.