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Bayesian virtual sensing in structural dynamics

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ABSTRACT

Structural monitoring and control utilize vibration measurements acquired by a sensor network. Combined empirical and analytical virtual sensing is introduced to estimate full-field dynamic response of a structure using a limited number of sensors. Bayesian empirical virtual sensing technique is developed to obtain less noisy estimates of sensor data. Then, analytical virtual sensing utilizes the expansion algorithm to compute the full-field response. If the sensor noise is known, virtual sensors are more accurate than the corresponding physical measurements with any number of sensors in the network. Often, the measurement error has to be estimated. The upper bound of the sensor noise variance is derived, and the effect of the noise estimation error on the accuracy of virtual sensors is studied. Numerical simulations are performed for a structure subject to unknown random excitation in order to validate the proposed virtual sensing algorithms. Displacement and strain sensor networks with different numbers of sensors and different noise models are studied.

1. Introduction

Sensors and monitoring systems appear everywhere. Current technology makes it possible to install wireless sensor networks with a high number of small low-cost sensor nodes. An important application is structural monitoring, which utilizes vibration measurements acquired by a sensor network to assess the condition of the structure. For example, fatigue assessment uses stress or strain histories at critical locations of the structure. However, some locations may be impractical or inaccessible for instrumentation. Therefore, accurate estimation of the response at these locations using available measurements would make fatigue assessment more reliable. Also, in structural health monitoring (SHM), the measurement accuracy is important in order to detect small damage and get an early warning to save costs or lives.

Virtual sensing techniques estimate quantities of interest using the available physical sensors. In vibration monitoring, the response of linear structures can be assumed to consist of modal contributions, in which only a few natural modes are active. With this assumption, a finite number of sensors is sufficient to make the sensor network redundant. This redundancy can be utilized in the development of virtual sensors.

Virtual sensing can be either analytical (model-based) or empirical (data-driven) [1]. Analytical virtual sensing techniques use information available from a limited set of physical sensors together with a finite element model to calculate an estimate of the quantity of interest. For example, it is possible to estimate the stress or strain field from acceleration measurements. Analytical mode shapes from the finite element model can be used as a basis to estimate the response at unmeasured locations by an expansion algorithm [2].

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Some challenges exist in analytical virtual sensing: For civil engineering structures, the excitation or other environmental loads are often difficult to measure, and the response is only measured. The estimation accuracy may suffer from the measurement errors, especially for wireless sensor networks (WSN) with low-cost hardware. In addition, the model errors, or more specifically the errors in the mode shapes, affect the accuracy of the virtual sensors.

Analytical virtual sensing in structural dynamics has been studied e.g. in the following papers. Avitabile [2] applied virtual sensing to correlation of analytical and experimental models using model expansion. Sestieri et al. [3] estimated rotational degrees of freedom (DOFs) from a limited set of measured translational DOFs. Hjelm et al. [4] estimated stress histories from acceleration measurements. Iliopoulos et al. [5] applied virtual sensing to fatigue assessment of wind turbines. Full-field dynamic stress/strain field was estimated from limited sets of measurement data in [6,7]. Papadimitriou et al. [8] introduced a technique that uses Kalman filter to estimate power spectral density of strain anywhere in the structure for fatigue assessment. Azam et al. [9] developed a dual Kalman filter for estimating the unknown input and states of a linear system. Maes et al. [10] compared three different response estimation algorithms: a joint input-state estimation algorithm, a Kalman filter algorithm, and a modal interpolation algorithm. Kullaa [11] applied dynamic substructuring to virtual sensing. Kalman filtering has also been used for signal estimation with singular systems corrupted by multiplicative noise using both instantaneous and delayed measurements [12]. A nice tutorial review of Kalman filtering techniques is provided in [13].

Empirical virtual sensing can be used to replace a temporarily installed or failed sensor [14]. Empirical virtual sensing has also been used for damage or sensor fault identification [15]. The accuracy of estimation can be increased by increasing the number of sensors in the network. For example, Kalman filtering has been used with correlated signals for noise reduction [16].

Integration of the acceleration signals, resulting in velocity or displacement, can also be categorized as virtual sensing. There are several studies addressing this problem. Relatively recent papers include [17], in which the displacement is estimated from an ill-posed boundary value problem solved using a regularization term. Another interesting application is using combined acceleration and strain measurements together with a numerical model of the structure to estimate displacements at low and high frequencies [18].

In this paper, a combined virtual sensing algorithm is introduced, which applies both empirical and analytical virtual sensing to obtain an estimate of the quantity of interest. The algorithm uses output-only data, in which the vibration response is only measured, while the excitation remains unknown. The objective of the combined method is to reduce uncertainty of virtual sensing techniques due to measurement error. A redundant sensor network is assumed. Although an exact finite element model is unrealistic, model error is ignored in this paper, making it possible to isolate the measurement error as the sole source of error thus facilitating the comparison of different algorithms. Also, errors due to double integration of accelerations are omitted for the same reason by presuming direct displacement measurements.

The paper is organized as follows. In Section 2, the theory of Bayesian empirical virtual sensing is derived and analytical virtual sensing is briefly introduced. A combination of empirical and analytical virtual sensing for more accurate full-field response is proposed. Also, an upper bound of the measurement error is derived. Numerical experiments are performed in Section 3 with noisy vibration measurements to validate the proposed method and to study the effect of the error of the noise estimate on the accuracy of the virtual sensors. Two different sensor networks and noise models are investigated. Concluding remarks are given in Section 4.

2. Virtual sensing in structural dynamics

Virtual sensing (VS), or soft sensing, is used to provide an alternative to physical measurement instrument. The quantity of interest is estimated using the available measurements and the system model. Virtual sensing can be classified into empirical and analytical techniques.

2.1. Bayesian empirical virtual sensing

Empirical virtual sensing is based on available historical measurements. These data are used to estimate the correlation between the measured quantities. Empirical virtual sensing uses regression techniques that can be implemented using different statistical methods, for example linear regression or neural network regression.

2.1.1. Measurement model

Consider a sensor network measuring p simultaneously sampled variables $\mathbf{y} = \mathbf{y}(t)$ at time instant t. Each measurement $\mathbf{y}$ includes measurement error $\mathbf{w} = \mathbf{w}(t)$:

$$\mathbf{y} = \mathbf{x}_m + \mathbf{w} \tag{1}$$

where $\mathbf{x}_m = \mathbf{x}_m(t)$ are the true values of the measured degrees of freedom. The objective is to find a less noisy estimate for the true values $\mathbf{x}_m$ utilizing the noisy measurements $\mathbf{y}$ from the sensor network.

2.1.2. Bayes' rule

From Bayes' rule,