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Numerical antiresonance continuation of structural systems

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ABSTRACT

Tuned dynamic absorbers are usually used to counteract vibrations at a given frequency. Presence of non-linearities causes energy-dependent relationship of their resonance and antiresonance frequencies at large amplitude of motion, which consequently leads to a detuning of the absorber from the targeted frequency. This paper presents a procedure to track an extremum point (minimum or maximum) of nonlinear frequency responses, based on a numerical continuation technique coupled to the harmonic balance method to follow periodic solutions in forced steady-state. It thus enable to track a particular antiresonance. The procedure is tested and applied on some application cases to highlight the resonance and antiresonance behavior in presence of geometrically non-linear and/or inertial interactions.

1. Introduction

In the widely investigated domain of attenuation of vibrations [1], passive tuned vibration absorbers are usually a suitable choice. To overcome the low robustness of linear absorbers, based on the Frahm's damper concept [2], nonlinear ones have been proposed. We can gather them into two families. The first one is the class of non linear tuned vibration absorbers (NLTVA). Those devices are based on the concept, introduced in [3], that the nonlinear absorber should possess a frequency-energy dependence identical to that of the nonlinear host system. Among other studies on the subject, [4] proposes a generalization of DenHartog's equal-peak method to non linear systems and gives a methodology to tune the whole nonlinear restoring force of the NLTVA on that of the primary structure of interest. The second class of absorbers is the one of so-called nonlinear energy sinks (NES) [5,6]. They exploit a secondary oscillator designed with strong non-linearities. Several ideas have bean proposed. Among others, one can cite essential cubic stiffness [7], non-polynomial non-linearities [8], vibro-impact devices [9].

For some applications, the nonlinear nature of the absorber is a constraint imposed by the design itself. For example, the so-called centrifugal pendulum vibration absorber (CPVA) used by the automotive industry to counteract irregularities of rotation of powertrains is intrinsically nonlinear. This passive device evolves in a centrifugal acceleration field and consists of a set of pendular oscillators (the CPVAs) acting as dynamic absorbers on a rotating primary structure [10–12]. Behavior of pendular oscillators is known to be sensitive to the path shape followed by their center of mass which involves geometric non-linearities at large amplitude of motion. Circular path causes a softening behavior while the so-called tautochronic path (from Greek: *tauto*, same, and *chrono*, time) keeps frequency independent of the motion amplitude [13]. For example, tautochrone curves in gravitational and centrifugal acceleration fields are respectively the cycloid [14] and the epicycloid [15].

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When CPVAs are coupled to the primary structure, the system is subject to strong nonlinear inertial coupling between host structure and absorbers. This additional source of non-linearities re-activates the frequency-energy dependence even if the oscillator has been designed to be tautochronous. The result is the detuning of the CPVA and the shifting of the optimal operating frequency (an antiresonance of the whole system) from the targeted tuning frequency. For such applications, the knowledge of the antiresonance behavior is essential.

The study of the response of a nonlinear dynamic system can be dealt with through analytical or numerical approaches. The first category is particularly suitable for a parametric study of systems having few degrees of freedom. One can cite the Lindstedt-Poincaré method or multiple time scales method [16] for example. Although those methods give an analytical approximation of the solution, most of them assume that the effect of the non-linearity is small, which becomes invalid for a large amplitude of motion. From a numerical point of view, there is a large variety of methods. Best known are direct time integration procedures of the governing equations. They are usually easy to implement and deployed in many commercial softwares. However, they may require extensive computation time to reach the steady state in case of long transient and don't give informations on the unstable solutions. An other class of numerical methods is the periodic steady-state analysis. One can cite the shooting method [17] that allows to directly reach a periodic steady-state through an iterative correction of the initial conditions up to the basin of attraction of periodic solutions. Some other methods are based on a discretization of the periodic solution through an expansion as a linear combination of known functions. For instance, the orthogonal collocation method adopts a piecewise polynomial representation of the solution [18]. In the frequency domain, the so-called harmonic balance method (HBM), used in this paper, is very popular and consists in discretizing the unknown solution by means of a truncated Fourier series [16].

In practice, it is often convenient to compute branches of steady-state periodic solutions as a function of a given parameter (amplitude, frequency, ...), leading to numerical continuation methods. Most common procedures are surely those based on a predictor corrector method with arc length parametrization, implemented for example in AUTO [19] or MATCONT [20] softwares. Another approach, the so-called asymptotic numerical method (ANM), addressed in the following of this paper, is based on a high order predictor method without correction and adopts an analytical representation of the branch of solution [21].

Prediction of an extremum of nonlinear frequency response is often helpful for the design of nonlinear systems. For instance, Habib and Kerschen use in Ref. [22] a perturbation method coupled to the HBM in order to analytically predict resonance peaks and establish some design rules about of the NLTVA. Some numerical methods are proposed to accurately predict amplitude resonance peak of a nonlinear system. For instance, Liao et al. use optimization procedure coupled to the HBM to obtain the worst periodic vibration response amplitude [23]. The case of quasi-periodic oscillations with uncertainties has been dealt in Ref. [24].

The contribution of this paper is to propose a general method to follow periodic solutions at a particular point of a dynamic response, namely an extremum (minimum as well as maximum) of a particular nonlinear frequency response of the system. Since an antiresonance can be defined as a particular minimum, our procedure is able to track antiresonances. It is also able to track resonance points, defined as a maximum of a frequency response. Our contribution provides an additional tool for the development of dynamic absorbers, in addition to particular procedures such as bifurcation tracking methods [25,26]. In this article, we have implemented our strategy of continuation of extrema in the framework of the ANM/HBM and the software Manlab2.0 [27], but since it is very close to fold bifurcation continuation, it could also be implemented in generic continuation solvers like AUTO [19] and MATCONT [20].

Stability of the solution is an essential aspect and must be considered during design phase of nonlinear systems. In particular, there can exist conditions where the desired response can be unstable near antiresonance [28,29]. However, the authors want to keep the focus of the paper on extremum tracking procedure and stability of the response is not addressed.

This paper is organized as follows: Section 2 introduces the framework of the study and the concept of antiresonance is extended from linear to nonlinear scope. The formulation of the antiresonance continuation is addressed in Section 3 and its numerical solving with the asymptotic numerical method in Section 4. Then, the methodology is applied on the so-called Euler's pendulum which exhibits non-linearities of various nature. Finally, the conclusion is outlined in the last section.

2. General framework and antiresonance concept

In this section, we establish the framework of the study through the highlighting of some fundamental concepts faced by N_{eq} degrees-of-freedom dynamical systems governed by:

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{D}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} + \boldsymbol{f}_{nl}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \boldsymbol{F}\cos\omega t, \tag{1}$$

where $\dot{(-)}$ stands for the derivative of $(-)$ w.r.t. the time t, $\boldsymbol{x}$ is the vector of unknowns, $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{D}$ is the damping matrix, $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{f}_{nl}(\boldsymbol{x}, \dot{\boldsymbol{x}})$ is the vector of nonlinear forces that depends on $\boldsymbol{x}$ and $\dot{\boldsymbol{x}}$ and $\boldsymbol{F}$ is the vector of external forces.

In order to illustrate the concepts introduced in this section, we consider the two degrees of freedom Duffing-like system depicted on Fig. 1. It is composed of two oscillators. The first one of mass m_1, subjected to an external force $f\cos\omega t$, is linked to the Galilean frame through a linear stiffness k_1 and a linear viscous damping c_1. Amplitude of oscillations of m_1 is measured by the degree of freedom x_1. The second oscillator, of mass m_2, is coupled to the primary one through a linear stiffness