Mechanical Systems and Signal Processing 116 (2019) 985–996

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

System-level modeling of nonlinear hysteretic piezoelectric actuators in quasi-static operations

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ARTICLE INFO

Article history:
Received 13 October 2017
Received in revised form 13 July 2018
Accepted 15 July 2018

Keywords:
Hysteresis
Actuator
Piezoelectric
Modeling

ABSTRACT

This paper aims at providing a simple yet efficient model for predicting the strain-voltage relationship in piezoelectric actuators subjected to high driving conditions, and featuring hysteretic response under such excitation. The originality of the model lies in proposing a high-level, system approach for modeling hysterestic response, using a butterfly-shaped electromechanical coupling term between the voltage and strain derivatives. The model is based on function reversal, and can be applied to many hysteresis types as the function can be properly chosen. Furthermore, the model requires few parameters and works both in unipolar and symmetric cycles, while being able to handle minor loops as well. Experimental results made on an unipolar piezoelectric actuator show good agreement with theoretical predictions. As a general hysteresis model, the proposed model can besides be applied to other domains, such as in magnetism, magnetic actuation or hydraulics.

1. Introduction

Along with the development and spreading of smart materials that allow converting energy from a physical domain (*e.g.*, mechanical or thermal) into electricity, a significant variety of applications have emerged or have taken advantage of such elements [1–6]. Among other well-known smart materials such as Shape Memory Alloys or thermoelectric modules, piezoelectric elements have recently attracted many attention from both scientific and industrial communities for a wide range of applications such as actuation, sensing, positioning or energy harvesting [7–13]. Such an interest is explained by the compactness of piezoelectric devices (in contrast to electromagnetic actuators for instance), high energy densities, high force and good integration potentials. However, as many other classes of actuators, piezoelectric devices show hysteretic behavior when driven at high voltage/electric field, due to the particular electric field vs. polarization response of the material, which has to be addressed for efficient actuator control [14–18].

Taking into account such a hysteresis can be done using many approaches, each having its own advantages and drawbacks [19]. Roughly speaking, hysteresis models can be classified into two categories. The first one relies on purely numerical approaches (usually phenomenological) and uses distribution of elementary loops ("*hysterons*") through state matrices or vectors to describe the behavior. This class of models includes the well-known Preisac model and its enhancements such as the Krasnosel'Skii-Pokrovskii (KP) method [20,22,21], as well as Maxwell-Slip approaches and similar dry

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https://doi.org/10.1016/j.ymssp.2018.07.025
0888-3270/

friction-based methods and their derivatives [23–26]. Although such models are very effective in accurately describing the hysteretic behavior, they may require significant computational efforts and memory size. In addition, for models based on phenomenological approaches (*e.g.*, Preisac), the link of the algorithm with physical mechanisms at the origin of hysteresis may be difficult to understand, although some works pointed out some possible explanations [21].

The second class of hysteresis models consists of using single state vectors or reduced parameter sets along with mathematical formulations. This includes Bouc-Wen, Jiles-Atherton and Duhem methods [27–32]. Thank to their reduced parameter sets, such methods are effective for implementation in controllers. However, they may show limitations in terms of robustness and may also fail in representing particular effects such as material/device anisotropy or minor loops (requiring model complexification to represent those).

However, from a high-level point of view, hysteresis between output quantity (*e.g.*, strain or displacement) and driving quantity (*e.g.*, voltage, current, temperature or pressure) is usually antisymmetric so that the output-input curve can be seen as shifted and reversed when going to the opposite way (change in the derivative sign). Therefore, the relationship between the output and driving quantities is generally symmetric. Hence, this paper proposes to model hysteretic behavior through a particular function applied to the multiphysic coefficient linking output quantity derivative and input quantity derivative. Contrary to the Duhem model [31,32], the proposed model relies on a single function and does not explicitly require separating increasing and decreasing rates of the input. Furthermore, the change in the device behavior due to a different input slope sign is simply obtained by a shift in the controlling function, and the output derivative in the exposed approach is solely dependent on the input and its derivative, so that stability can be expected to be ensured. Finally, the function identification process in the Duhem model may be quite cumbersome, while in the proposed approach a single experimental curve and one low-voltage measurement are sufficient. This therefore allows setting up a simple model requiring very few parameters, easy identification and simple algorithm, that can therefore easily and efficiently be implemented in controllers for instance without the need of heavy memory requirements and computational efforts.

The paper is organized as follows. Section 2 aims at exposing the origin of the model, as well as proposing a basic mathematical theory to predict actuator response, which is then extended in order to better reflect the device behavior. Proposals in terms of actual implementation are also devised in Section 3. Then, Section 4 aims at validating the model through comparison with experimental measurements made on a piezoelectric actuator driven at relatively high voltage. Finally, Section 5 proposes to summarize the main findings of the paper, along with some perspectives in model refinement.

2. Nonlinear hysteretic actuator modeling

When dealing with piezoelectric actuators far below their resonance (*i.e.*, quasi-static regime), the general considered linear equation between the strain S and the piezovoltage V is given by:

$$S = \alpha V \tag{1}$$

where α denotes the strain/voltage relationship coefficient and depends on structural and material properties (for instance dimensions, elastic rigidity and piezoelectric charge constant). However, when dealing with high solicitations, linear constitutive equations are no longer valid, which can be reflected by introducing a voltage-dependence in the strain/voltage coefficient so that Eq. (1) may be rewritten to:

$$S = \int \alpha(V)\mathrm{d}V \tag{2}$$

More particularly, for hysteretic systems, the evolution of $\alpha(V)$ as a function of V describes a minor butterfly loop inside the major loop of the material itself, as depicted in Fig. 1.

2.1. Basic model principles

Therefore, being able to describe the actuator butterfly loop (corresponding to a minor loop in the global material response – Fig. 1) in the plane (V, α) would allow reconstructing the hysteresis response using Eq. (2). To do so, the modeling of the parameter $\alpha(V)$ usually consists in inverting the response to an increasing (*resp.* decreasing) voltage with respect to the x-axis. However, such an approach, in addition to be complex and computationally intensive, is limited to particular operating conditions (such as constant voltage magnitude and symmetric cycles) and cannot easily adapt to complex excitations. Hence, instead of relying on the controlled parameter inversion and shift, it is proposed here to use an inversion and shift of the output/input coefficient.