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Localization of dynamic forces on structures with an interior point method using group sparsity

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ABSTRACT

This paper presents an algorithm for the localization of forces applied to a structure. The force estimation is performed in the frequency domain based on a limited number of sensors and a linear dynamic system model and it involves minimizing an objective function penalized with a group sparsity term. The minimization of this objective function is formulated as a second order cone program, which is solved using an interior point method. This allows for a reduction of the calculation time when compared to other algorithms that are currently available to enforce group sparsity on the forces, especially for large scale problems. The presented algorithm is first verified using numerical simulations. Next, a validation is performed using data obtained from a field test on a footbridge, where two locations on the bridge deck are excited using hammer impacts and the force localization is performed assuming a total of 108 possible force locations.

1. Introduction

For many civil engineering structures, it is of great importance to have information on the dynamic loads exciting the structure. In many cases, however, the loads cannot be measured directly. For example, applying force transducers might disturb the load, or the load of interest might excite the structure at an unreachable location or a location that is not known in advance. To overcome this problem, force identification techniques can be used, which combine the response of the structure measured at accessible locations with a dynamic system model to obtain an estimate of the dynamic loads.

Most force identification algorithms presented in the literature assume the location of the forces to be known and the number of forces to be smaller than or equal to the number of response measurements. The dynamic forces can then be estimated in the time domain [1–3] or, alternatively, in the frequency domain [4]. In the frequency domain, the Fourier transform of the forces is often obtained from the Fourier transform of the measured response through the pseudo-inverse of the frequency response function (FRF) matrix. In many cases, however, the ill-posedness of the force identification problem results in large estimation errors. Therefore, regularization techniques, such as penalizing the ℓ_2 norm of the force vector, are commonly used to improve the force estimation [4].

Different algorithms have been presented in the literature to solve the force localization problem. Guillaume et al. [5] developed an algorithm to localize the forces using a weighting matrix that is determined by minimizing the sum of the

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ℓ_p norm of the forces for all frequencies. The forces can then be determined by computing a weighted pseudo-inverse of the FRF matrices. The value of p is generally taken very small to enforce sparse solutions of the force identification problem. Ginsberg and Fritzen [6] proposed an ℓ_1 regularized least squares problem that penalizes force vectors with a large ℓ_1 norm to localize forces in the time domain. Problems of this type are commonly solved using iterative shrinkage-thresholding algorithms (ISTA) such as FISTA [7] or MTWIST [8]. Qiao et al. [9], however, proposed the use of interior point methods to solve this regularized problem. Kirchner et al. [10] recently developed the compressive sensing-moving horizon estimator (CS-MHE) which combines a moving horizon estimator (MHE) with an ℓ_1 regularization term. This allows for a recursive solution of the force localization problem in the time domain. Aucejo [11] developed the GIRLS algorithm to solve a similar regularized least squares problem at each frequency and recently suggested an alternative technique based on multiplicative regularization, where the regularization parameter and the forces are determined simultaneously [12]. Rezayat et al. [13] developed the G-FISTA method that minimizes a mixed ℓ_2/ℓ_1 regularized objective function for multiple frequencies at once. G-FISTA is an extension of FISTA [7] and allows to take into account a coupling between frequencies, i.e. if a location is excited by a force at a given frequency, then this force most likely also excites the same location at other frequencies.

This paper presents an alternative ℓ_2/ℓ_1 regularized objective function that is minimized to localize forces in the frequency domain. It is shown that this objective function can be cast in a second order cone program (SOCP), which can be solved using interior point methods. The presented technique is verified with a numerical example of a cantilever steel beam and validated using data obtained from a field test on a footbridge, showing the applicability of the method for a real-life case.

The outline of the paper is as follows. Section 2 presents an algorithm to localize forces and to estimate their narrow band frequency spectrum. Next, Section 3 shows an illustration of the algorithm based on numerical simulations for a cantilever steel beam, and compares the calculation time of the proposed technique to the calculation time of G-FISTA. Section 4 presents a validation of the algorithm using data obtained from a field test on a footbridge. Finally, in Section 5, the work is concluded.

2. Mathematical formulation

This section shows how forces can be identified in the frequency domain using the least squares method and how this commonly used method can be adapted in case the number of response measurements is smaller than the number of forces.

2.1. Force identification

Consider the vector $\mathbf{d}(\omega) \in \mathbb{C}^{n_\mathrm{d}}$ containing the Fourier transform of n_d measured outputs at a frequency ω:

$$\mathbf{d}(\omega) = \mathbf{H}_{\mathrm{dp}}(\omega)\mathbf{p}(\omega) + \mathbf{v}(\omega) \tag{1}$$

where $\mathbf{p}(\omega) \in \mathbb{C}^{n_\mathrm{p}}$ is the vector containing the Fourier transform of n_p forces, and $\mathbf{H}_{\mathrm{dp}}(\omega) \in \mathbb{C}^{n_\mathrm{d}\times n_\mathrm{p}}$ is the transfer function matrix that relates the force vector to its response. The vector $\mathbf{v}(\omega) \in \mathbb{C}^{n_\mathrm{d}}$ represents the error which in absence of modeling errors accounts for measurement noise and the response due to additional unknown forces that are not included in the force vector $\mathbf{p}(\omega)$. The aim of force identification is to determine the force vector $\mathbf{p}(\omega)$ from the response $\mathbf{d}(\omega)$, assuming the FRF matrix $\mathbf{H}_{\mathrm{dp}}(\omega)$ to be known.

In general, the force vector $\mathbf{p}(\omega)$ at a given frequency ω can be estimated by solving the following least squares problem:

$$\hat{\mathbf{p}}(\omega) = \arg\min_{\mathbf{p}(\omega)} \left|\left|\mathbf{T}^{-1}(\omega)\left(\mathbf{d}(\omega) - \mathbf{H}_{\mathrm{dp}}(\omega)\mathbf{p}(\omega)\right)\right|\right|_2^2 \tag{2}$$

where $\hat{\mathbf{p}}(\omega) \in \mathbb{C}^{n_\mathrm{p}}$ is the estimate of the force vector and $\mathbf{T}(\omega) \in \mathbb{C}^{n_\mathrm{d}\times n_\mathrm{d}}$ is a weighting matrix. In order to obtain the best linear unbiased estimate (BLUE) of $\mathbf{p}(\omega)$, the weighting matrix $\mathbf{T}(\omega)$ is calculated from the power spectral density function (PSD) $\mathbf{S}_{\mathrm{vv}}(\omega) \in \mathbb{C}^{n_\mathrm{d}\times n_\mathrm{d}}$ of the error $\mathbf{v}(\omega)$ as $\mathbf{S}_{\mathrm{vv}}(\omega) = \mathbf{T}(\omega)\mathbf{T}^*(\omega)$, where $\square^*$ denotes the Hermitian transpose. As such, the unique solution of this weighted linear least squares problem is the BLUE of $\mathbf{p}(\omega)$ in the absence of modeling errors and in case all other errors have a zero mean [14]. Note that the weighting implies that the PSD of the errors $\mathbf{S}_{\mathrm{vv}}(\omega)$ is known. For practical applications, this means that it should be determined from prior measurements on the structure in absence of the force vector itself (see e.g. [15]). It is additionally assumed that the PSD matrix of the errors $\mathbf{S}_{\mathrm{vv}}(\omega)$ is of full rank, which makes the Cholesky decomposition unique [16].

In case the number of response measurements is larger than or equal to the number of forces, the optimization problem in Eq. (2) is strictly convex and its unique solution is given by:

$$\hat{\mathbf{p}}(\omega) = \left(\mathbf{T}^{-1}(\omega)\mathbf{H}_{\mathrm{dp}}(\omega)\right)^{\dagger}\mathbf{T}^{-1}(\omega)\mathbf{d}(\omega) \tag{3}$$

where $\square^\dagger$ denotes the Moore–Penrose pseudo-inverse.