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An efficient method based on Bayes' theorem to estimate the failure-probability-based sensitivity measure

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ABSTRACT

The failure-probability-based sensitivity, which measures the effect of input variables on the structural failure probability, can provide useful information in reliability based design optimization. The traditional method for estimating the failure-probability-based sensitivity measure requires a nested sampling procedure and the computational cost depends on the total number of input variables. In this paper, a new efficient method based on Bayes' theorem is proposed to estimate the failure-probability-based sensitivity measure. The proposed method avoids the nested sampling procedure and only requires a single set of samples to estimate the failure-probability-based sensitivity measure. The computational cost of the proposed method does not depend on the total number of input variables. One numerical example and three engineering examples are employed to illustrate the accuracy and efficiency of the proposed method.

1. Introduction

Uncertainties occurring in the engineering systems and computer models [1–4] will lead to uncertain performance. To evaluate the degree of confidence of the model results and assess the risk, uncertainty analysis has been successfully and widely applied in engineering [5,6]. However, most applications of uncertainty analysis do not provide information on how the uncertainty of model output can be apportioned to the uncertainty of model inputs [7–9]. Then, researchers cannot decide how to allocate resources to input variables so as to reduce the uncertainty of model output most effectively [10,11]. Sensitivity analysis is a scientific tool which can help researchers understand "*how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input*" [12]. Local sensitivity analysis (LSA) is generally based on the derivative and is only informative at the nominal point where the derivative is computed. However, global sensitivity analysis (GSA) can measure the effect of input variables in their entire distribution ranges on the output [13–16]. GSA is also known as uncertainty importance analysis, which is used to identify what are the most critical and essential contributors to output uncertainty and risk [17]. Based on the results of GSA, researchers can reduce the uncertainty of output effectively through allocating more resources (e.g. people, time, financial budget, etc.) to the most important input variables and simplify the model by fixing non-important input variables at nominal values. A wide range of GSA methods have been proposed in the last several decades. For instance, screening method [18–20] was proposed for the cases where there are a lot of input variables and few model assumptions. Variance-based method [21–25] was proposed as a quantitative method which mainly focuses on the variance of model output. Later, the moment-independent method

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[26–28] was proposed to measure the effect of input variables on the whole distribution of model output. More details of GSA can be found in the reviews about sensitivity analysis [29,30].

In structural reliability analysis, the sign of model output representing whether the structure fails or not is of most interest [31]. The model output is then generally regarded as a binary state variable. The above GSA methods focus mainly on the real-valued continuous model output and could not be used in reliability analysis directly [32]. Since failure probability is of most interest in structural reliability analysis, most of the traditional reliability sensitivity measures are based on the partial derivative of failure probability with respect to the distribution parameters of input variables [33–36]. These sensitivity measures are local since the partial derivative is calculated at given nominal points of distribution parameters, and they can only reflect the effect of input variables on the failure probability at nominal values of input variables. To measure the overall effect of input variables on the failure probability of the model in their entire uncertainty domain, Cui et al. [37] proposed a failure-probability-based global sensitivity measure. This sensitivity measure is analogous to the moment-independent sensitivity measure proposed by Borgonovo [27], but it focuses on the failure probability which is often related to the tail behavior of the distribution of model output. In reliability analysis and design, this sensitivity measure can help find which of the input variables have significant effect on the failure probability.

The failure-probability-based sensitivity measure is defined as the average difference between the unconditional failure probability and the conditional failure probability on the input variable. It can reflect the average changes of the failure probability when the input variable can be fixed. The traditional method for estimating the failure-probability-based sensitivity measure requires a nested sampling procedure, in which the conditional failure probability must be calculated for different realizations of input variables. In this paper, a new efficient method based on Bayes' theorem is proposed to estimate the failure-probability-based sensitivity measure. In the proposed method, the failure-probability-based sensitivity measure can be rewritten as the area difference between the unconditional probability density function (PDF) of input variable and the conditional PDF of input variable on the failure domain. Therefore, estimating the conditional failure probability is avoided in the proposed method and it only requires estimating the conditional PDF of input variable on the failure domain. In this strategy, the failure-probability-based sensitivity measure is estimated with a single set of samples and the nested sampling procedure is avoided. The proposed method has a similar idea to generalized sensitivity analysis proposed Spear and Hornberger [38]. In the generalized sensitivity analysis, the model output is separated into two classes based on a given rule and the sensitivity of input variable is measured through the difference between the conditional distributions of input variable for different classes. In this paper, we actually divide the model output into two classes based on its sign. Then we can easily estimate the failure-probability-based sensitivity measure through the difference between the unconditional PDF of input variable and the conditional PDF of input variable on the failure domain.

The rest of this paper is organized as follows. Section 2 gives a brief review of the failure-probability-based sensitivity measure. Section 3 presents the new efficient method to estimate the failure-probability-based sensitivity measure. In Section 4, several examples are presented to illustrate the accuracy and efficiency of the proposed method. Section 5 gives the conclusion.

2. Review of the failure-probability-based sensitivity measure

2.1. Definition of the failure-probability-based sensitivity measure

Let $\mathbf{X} = (X_1, \ldots, X_d)$ be the vector of input random variables. The joint PDF of $\mathbf{X}$ is denoted by $f_{\mathbf{X}}(\mathbf{x})$. The marginal PDF of X_i is denoted by $f_{X_i}(x_i)$ $(i = 1, \ldots, d)$. When all the input variables are independent, $f_{\mathbf{X}}(\mathbf{x}) = \prod_{i=1}^{d} f_{X_i}(x_i)$. The output variable Y is defined by $Y = g(X_1, \ldots, X_d)$, where $g(X_1, \ldots, X_d)$ is the performance function. Let F denote the failure of structure. The failure probability $P(F)$ is defined by $P(F) = P\{g(\mathbf{X}) \leqslant 0\}$.

When the input variable X_i is fixed at a certain value x_i, the conditional failure probability of structure can be defined by $P(F|x_i) = P\{g(\mathbf{X}) \leqslant 0|x_i\}$. The effect of the fixed value x_i of input variable X_i on the failure probability can be measured by the difference between $P(F)$ and $P(F|x_i)$, i.e.

$$s(x_i) = |P(F) - P(F|x_i)|. \tag{1}$$

$s(x_i)$ is only a function of x_i. Since x_i is just a certain value of random variable X_i with PDF $f_{X_i}(x_i)$, the average effect of X_i on the failure probability can be expressed by the expectation of $s(x_i)$, i.e.

$$E_{X_i}[s(x_i)] = E_{X_i}|P(F) - P(F|x_i)| = \int_{X_i} |P(F) - P(F|x_i)| f_{X_i}(x_i)\mathrm{d}x_i. \tag{2}$$

In order to get a sensitivity measure lying between 0 and 1, a normalized failure-probability-based sensitivity measure η_i for input variable X_i was proposed by Cui et al. [37] as

$$\eta_i = \frac{1}{2} E_{X_i}|P(F) - P(F|x_i)| = \frac{1}{2}\int_{X_i} |P(F) - P(F|x_i)| f_{X_i}(x_i)\mathrm{d}x_i\ . \tag{3}$$

The sensitivity measure η_i is analogous to the moment-independent sensitivity measure δ_i proposed by Borgonovo [27]. The moment-independent sensitivity measure can measure the average effect of input variable on the whole probability