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Brief paper

Global stabilization of periodic linear systems by bounded controls with applications to spacecraft magnetic attitude control☆

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ABSTRACT

We study in this paper the three-axis magnetic attitude control of small spacecraft by considering the actuator saturation. By noticing that the linearized dynamics is a neutrally stable linear periodic system, a general problem of global stabilization of linear periodic systems with bounded feedback is formulated and solved by both state feedback and observer based output feedback. Explicit saturated linear feedback control laws are established by using solutions to some Lyapunov differential equations associated with the open-loop systems and global stability of the closed-loop system is proved by constructing an explicit Lyapunov function. To apply the theoretical results on the three-axis magnetic attitude control systems, necessary and sufficient conditions are established to guarantee that the linearized open-loop system is neutrally stable and explicit solutions to the associated Lyapunov equations are obtained consequently. The controllers for the three-axis magnetic attitude control systems are then given in closed-form. Simulations show the effectiveness of the proposed approach.

1. Introduction

Many spacecraft possess attitude stabilization systems since they are required to maintain a certain prescribed attitude in the space during their useful life (Gerlach, 1965). Typical attitude stabilization control systems include passive spin-stabilized systems, gravity-gradient stabilized systems and magnetic torquer stabilized control systems (Psiaki, 2001). It has been realized that the most accurate designs normally include momentum wheels and/or reaction wheels. For the magnetic torquer attitude stabilization control system, the only actuators are the magnetic torque rods, which weigh less than both a gravity-gradient system and a wheel-base system, and will use less power than a wheel-based system (Psiaki, 2001). For these advantages, the magnetic torquer attitude control system is recommended for small spacecraft applications since the weight and power budgets for small spacecraft are very restricted (Psiaki, 2001).

The main feature of the magnetic torquer attitude stabilizing system is that the dynamics of the system varies periodically with the period equaling the orbit period of the spacecraft. As a result, time-varying controllers are generally necessary. There have been a great number of literature that deal with the attitude stabilization by magnetic torquer control. Particularly, these existing literature have utilized properly the achievements in the control of linear periodic systems. To mention a few, the H_∞ controller for periodic linear systems is utilized in Kulkarni and Campbell (2004) to solve the three-axis magnetic attitude stabilization problem via discretization of the continuous-time plant; a weighted PD approach is proposed in Wood and Chen (2013) to overcome the classical torque-projection approach by minimizing a simple cost function; the linear quadratic regulator (LQR) based no-wheel controllers were designed in Guelmana, Wallera, Shiryaeva, and Psiakib (2005) for three-axis magnetic attitude stabilization systems with the magnetic torquers as the only actuators; time-varying periodic controllers are proposed in Findlay, de Ruiter, Forbes, and Lee (2013) and Lovera, De Marchi, and Bittanti (2002) by solving periodic Riccati differential equations for the attitude stabilization systems with respectively three-axis magnetic torquers and both magnetic torquers and momentum wheel; and a linear time-varying passivity-based approach was built in Forbes and Damaren (2011) for the attitude control systems design by employing both magnetic and mechanical actuators. See Pittelkau (1993), Psiaki (2001), Reyhanoglu and Hervas (2011), Silani and Lovera (2005),

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Tsiotras (1994), Wood and Chen (2013) and the references therein for more related work on the related problems.

Constraints on the coils' magnetic dipoles play a major role in the formulation of the magnetic attitude control problem since magnetic coils can be only driven with limited currents, so that saturation of the control inputs (magnetic dipoles) can be an issue, particularly when dealing with large attitude errors and angular rates (Silani & Lovera, 2005). To the best of our knowledge, only a few papers are concerned with the magnetic torquer attitude stabilization problem by considering actuator saturation. To take the limited magnetic dipoles into consideration, a saturated periodic LQR controller multiplied by a large parameter is proposed in Psiaki (2001) by utilizing the infinite gain margin of the LQR controller. The resulting closed-loop system can only claimed to be locally stable. A nonlinear small feedback was established in Lovera and Astolfi (2004) (without considering the effect of gravity gradient torques in the controller design) to solve the global attitude stabilization problem for an inertial pointing spacecraft with magnetic actuators. Since the magnetic torquer attitude stabilization with bounded feedback is an example of stabilization of linear periodic systems with bounded feedback, the theory built for the later problem can be utilized to solve the former one. However, different from the control of linear time-invariant systems with bounded feedback which have been extensively studied in the literature (see, for example, Chen, Ge, & Ren, 2011; Hu & Lin, 2001; Huang & Lam, 2002; Lin & Saberi, 1993; Liu & Tong, 2015 ; Sussmann, Sontag, & Yang, 1994; Ye & Gui, 2012 and Zhou, Lin, & Duan, 2011), for the stabilization of linear periodic systems with bounded controls, especially, for the global and semi-global stabilization, there are only a few results available in the literature. This is because the control of linear (periodic) time-varying systems even without considering control constraints is a challenging problem (see, for example, Bittanti & Colaneri, 2008; Savkin & Petersen, 1998; Zhou & Duan, 2012). Only very recently has the semi-global stabilization of linear periodic systems with bounded controls been solved in Zhou, Hou, and Duan (2013) for continuous-time systems and in Zhou, Duan, and Lin (2011) for discrete-time systems.

We study in this paper the three-axis magnetic attitude control of small spacecraft by considering the magnitude constraints in the actuators. A more general theoretical problem is solved by constructing explicit controllers and the obtained theoretical results are then applied on the three-axis magnetic attitude control systems design. There are four contributions of this paper. First, a general problem of global stabilization of neutrally stable linear periodic systems with actuator saturation is completely solved by both saturated linear state and observer based output feedback, and the global stability of the closed-loop system is proved by constructing explicit Lyapunov functions. Second, necessary and sufficient conditions are derived for guaranteeing that the open dynamics of the three-axis magnetic attitude control system is neutrally stable, namely, all the eigenvalues of the system matrix are on the imaginary axis and simple. Third, explicit solutions to the Lyapunov matrix equation (inequality) associated with the (open-loop) linearized three-axis magnetic attitude control system are obtained and thus explicit stabilizing controllers are established. Finally, the simulations of the closed-loop system consisting of the proposed saturated linear feedback and the original nonlinear plant show that the proposed controller can achieve attitude stabilization for large initial conditions and are superior to some existing approaches in terms of control performances.

The remainder of this paper is organized as follows. Section 2 states the problem to be solved in this paper. Section 3 solves the main problem and the obtained theoretical results are applied on the three-axis spacecraft magnetic attitude control systems design in Section 4. The paper is concluded in Section 5.

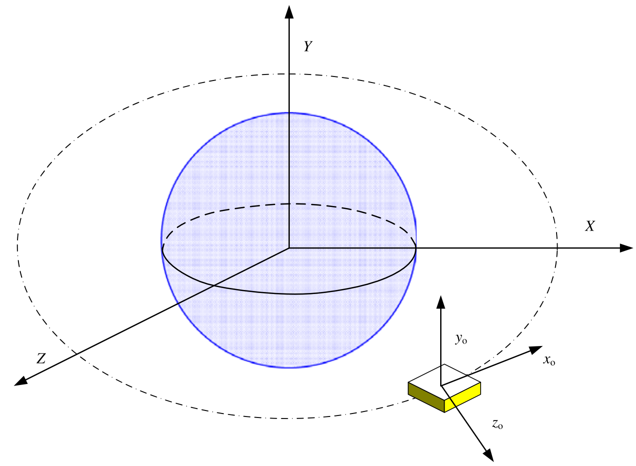

Fig. 1. The geocentric inertial frame (XYZ) and the orbital reference frame ($x_o y_o z_o$).

2. Problem motivation and formulation

2.1. Problem motivated by the spacecraft magnetic attitude control

Let X–Y–Z be the geocentric equatorial inertial reference frame denoted by $\mathfrak{F}_i$, where X axis points in the vernal equinox direction, the X–Y plane is the Earth's equatorial plane, and the Z axis coincides with the Earth's axis of rotation and points northward. Let $\mathfrak{F}_b$ be the spacecraft-fixed body frame and $\mathfrak{F}_o$ be the orbital reference with the origin at the center of mass of the spacecraft, x_o being along the orbit direction, y_o being perpendicular to the orbit plane and z_o being in the nadir direction. The orientation of the spacecraft is described relative to the orbital frame $\mathfrak{F}_o$. It follows that if the attitude of the satellite is the identity, the body coordinates $x_b - y_b - z_b$ coincide with exactly the orbital coordinates $x_o - y_o - z_o$ (Reyhanoglu & Hervas, 2011; Wertz, 1978). See Fig. 1 for an illustration.

The orientation of $\mathfrak{F}_b$ relative to $\mathfrak{F}_o$ can be given by the attitude matrix Ψ, which is parameterized by the quaternion as Psiaki (2001), Reyhanoglu and Hervas (2011) and Wertz (1978)

$$\begin{aligned}\Psi &\triangleq \begin{bmatrix}\varphi_x & \varphi_y & \varphi_z\end{bmatrix}\\ &\triangleq \begin{bmatrix} 1-2\left(q_2^2+q_3^2\right) & 2(q_1q_2+q_3q_4) & 2\left(q_1q_3-q_2q_4\right)\\ 2\left(q_1q_2-q_3q_4\right) & 1-2\left(q_1^2+q_3^2\right) & 2\left(q_2q_3+q_1q_4\right)\\ 2\left(q_1q_3+q_2q_4\right) & 2\left(q_2q_3-q_1q_4\right) & 1-2\left(q_1^2+q_2^2\right)\end{bmatrix},\end{aligned}$$

in which $q = [q_1, q_2, q_3, q_4]^{\mathrm{T}}$ is the quaternion. Let $\omega_0 = \sqrt{\frac{\mu}{r^3}}$ denote the orbital rate, where $\mu = 3.986 \times 10^{14}$ m^3/s^2 is the gravity constant and r is the semimajor axis of the orbit.

The attitude kinematics can be written in terms of quaternions as Psiaki (2001), Reyhanoglu and Hervas (2011) and Wertz (1978)

$$\dot{q} = \frac{1}{2}\begin{bmatrix} 0 & \omega_{rz} & -\omega_{ry} & \omega_{rx}\\ -\omega_{rz} & 0 & \omega_{rx} & \omega_{ry}\\ \omega_{ry} & -\omega_{rx} & 0 & \omega_{rz}\\ -\omega_{rx} & -\omega_{ry} & -\omega_{rz} & 0\end{bmatrix} q, \tag{1}$$

where $\omega_r = \left[\omega_{rx}, \omega_{ry}, \omega_{rz}\right]^{\mathrm{T}}$ is the (relative) angular velocity of body frame $\mathfrak{F}_b$ relative to the orbital frame $\mathfrak{F}_o$. The attitude dynamics can be expressed as Psiaki (2001), Reyhanoglu and Hervas (2011) and Wertz (1978)

$$\begin{cases} J_x\dot{\omega}_x + \left(J_z - J_y\right)\omega_y\omega_z = T_{gx} + T_{mx},\\ J_y\dot{\omega}_y + \left(J_x - J_z\right)\omega_x\omega_z = T_{gy} + T_{my},\\ J_z\dot{\omega}_z + \left(J_y - J_x\right)\omega_y\omega_x = T_{gz} + T_{mz}, \end{cases} \tag{2}$$

where J_x, J_y and J_z are the inertia of the spacecraft, $\omega = [\omega_x, \omega_y, \omega_z]^{\mathrm{T}}$ is the angular velocity of the body frame $\mathfrak{F}_b$ with respect