Mechanical Systems and Signal Processing 114 (2019) 1–24

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

On choice and effect of weight matrix for response sensitivity-based damage identification with measurement and model errors

Zhong-Rong Lu, Junxian Zhou, Li Wang *

Department of Applied Mechanics and Engineering, Sun Yat-sen University, Guangzhou, PR China

A R T I C L E I N F O

Article history:
Received 3 June 2017
Received in revised form 17 April 2018
Accepted 1 May 2018

Keywords:
Optimal weight matrix
Structural damage identification
Measurement and model errors
Enhanced response sensitivity approach
Hybrid data

A B S T R A C T

This paper aims to present a thorough view on the choice and effect of the weight matrix for response sensitivity-based damage identification with measurement and/or model errors. The derivation of the optimal weight matrix is mainly twofold. On the one hand, when only measurement errors are involved, the optimal weight matrix is found to be inverse proportional to the measurement error covariance by minimizing the expectation of squares error of the whole identification results. On the other hand, if model errors are additionally considered, the optimal weight matrix then depends not only on the measurement error covariance, but also on the model error covariance. Further analysis reveals that the optimal weight matrix can also make the 'relative error'—square-root of expectation of squares error in every individual damage parameter minimized. Then, the effect of the proposed optimal weight matrix with measurement and/or model errors is studied on two typical examples—a plane frame and a simply-supported plate. Results show that when hybrid types of measurement data—accelerations, displacements and/or eigenfrequencies are used or when the response data is sensitive to model errors, the optimal weight matrix should be invoked to get reasonably good identification results and the improvements brought by the optimal weight matrix are substantial. The whole work shall be instructive for damage identification when different types of measurements are available and when model errors are non-negligible.

1. Introduction

In-service structures are always working under environmental aggressions, possibly unexpected loads and material creep *etc*, and as a consequence, damage or degradation of structural stiffness may occur which is harmful for practical use. To avoid this, the structural health monitoring (SHM) system [1] is necessarily introduced for the sake of preventive monitoring, detection and maintenance. Indeed, the central ingredient of the SHM system resides in the structural damage identification.

Generally, damage is embodied as the reduction of structural stiffness and therefore, the main task of structural damage identification is to inversely identify the damage locations and intensities (or stiffness reductions) from the measured data, e.g., acceleration responses, displacement responses, eigenfrequencies and/or eigen modes of a structure. Typically, such a procedure belongs to a class of inverse problems and is often modeled as a nonlinear (weighted) least-squares optimization

---

* Corresponding author.
E-mail address: wangli75@mail.sysu.edu.cn (L. Wang).

https://doi.org/10.1016/j.ymssp.2018.05.007
0888-3270/

problem [2,3]. Various methods including the genetic-like algorithms [4–6] and the gradient-based approaches [2,3,7] have been developed to solve such a nonlinear least-squares optimization problem, among which the hessian-free gradient-based minimization techniques such as the Gauss-Newton method, the Levenberg-Marquardt method [19] and the sensitivity-based approaches [4] are often preferred. In this paper, the sensitivity-based approaches [3,4,7–11] are used to solve the nonlinear least-squares optimization problem under the following considerations,

- Structural damage identification (or finite element model updating) is often accomplished with a large number of unknowns and therefore, the genetic-like algorithms with random search nature [5,6] are often not suitable for this problem due to the prohibitively high computational cost;
- Among the gradient-based approaches, the commonly used Newton method would require more complex second-order sensitivity analysis and more computational efforts [3] than the sensitivity-based approaches for which only the (first-order) sensitivity analysis should be conducted; this indicates that the Newton method is also inappropriate for the optimization problem;
- Recently, the sensitivity-based approaches along with proper regularizations or equivalent trust-region constraints have been shown to be weakly convergent [3].

In principle, the success of the sensitivity-based approaches lies in the sensitivity analysis and usage of regularization techniques [3,14]. On the one hand, different types of the measured data often correspond to different models [12] of the structure; for instance, time-domain data corresponds to the dynamic equation while eigenfrequencies and eigen modes are derived from modal analysis. On the other hand, different models would lead to quite different sensitivity analysis [2] and therefore, the sensitivity-based approaches should be adjusted with respect to the measurement types. Over the years, considerable work has been focused on the development and application of the sensitivity-based approaches for damage identification in various kinds of structures and with various types of the measured data. Classically, the modal data including eigenfrequencies and eigen modes is most used for damage identification. Farhat and Hemez [7] proposed to use eigen modes and the sensitivity analysis was found to be conducted in an element-by-element manner. Later, Bakir et al. [9] well identified the stiffness parameters from the modal data by incorporating the trust-region algorithm into the sensitivity approach. As is noteworthy, complete data of an arbitrary mode is required for the above two approaches which is often unavailable in many test cases. To this end, Chen and Maung [8] developed a regularized sensitivity approach for finite element model updating with incomplete modal data. In practice, the modal data including eigenfrequencies and incomplete modes is easily accessed and this explains why substantial attention was paid to damage identification or finite element model updating with the modal data in the literature. However, high-frequency information is often lost when experimental modal data is utilized for damage identification and this means that the modal data is not very suited for detecting and quantifying localized small size damage [12]. In contrast, as pointed out by Link and Weiland [12], time domain response data from impact tests carries high-frequency information and is sensitive to damage even if the damage is local [15]; this indicates that time domain response data is beneficial for the detection of local damage. Nevertheless, few publications [3,10–12] were found until recently addressing time domain response data in conjunction with the sensitivity analysis for damage identification. Lu and Law [10] proposed to use response sensitivity analysis for inverse damage identification, and later, Lu and Wang [3] enhanced the response sensitivity approach so that even large damage can be identified; Link and Weiland [12] conducted an elaborate comparative study for sensitivity-based damage identification using modal data and using time domain response data, for which the latter was shown to be superior in damage identification.

In this paper, the time domain response sensitivity approach [3] for damage identification is followed up for further study. The eigenfrequency data which is easily available may be additionally used to explore the possible improvement in damage identification with hybrid data [13]. As is mentioned above, the objective function of damage identification (or the optimization problem) is in the weighted least-squares form for which the residual between the measured data and the implicitly derived data and a weight matrix are involved. The sensitivity analysis is conducted regarding the residual term, while limited attention has been paid to the choice and effect of the weight matrix for the weighted nonlinear least-squares optimization problems. Notwithstanding, the weight matrix shall be carefully selected. On the one hand, different kinds of data—displacement, velocity, acceleration or even eigenfrequency correspond to different dimensions, e.g., m/s^2 for acceleration and m for displacement where m represents length in 'meter' and s time in 'second'. Without properly defined weight matrix, equivalent replacement of dimensions, e.g., m replaced by 10^3 mm (or millimeter) may lead to quite different objective functions and different identification results thereof. To avoid this, the scaling effect behind the weight matrix should be implicated. On the other hand, when the measured data is at different levels of errors, the weight matrix should be chosen such that large error corresponds to less weight. Primitively, there have already been some reasonable choices of the weight matrix [2,16–18], such as being the reciprocal of the covariance matrix of the measured data. Particularly, in sensitivity-based model updating, proper choice of the weight matrix in each updating step may even pose the equivalent effect of regularization [2,17].

In this paper, the main aim is to present a thorough view on the optimal choice and the specific effect of the weight matrix for response-sensitivity-based damage identification. Distinct to the work in [2,17], the weight matrix herein for the nonlinear least-squares optimization problem does not change during the iterative identification procedure and does not intend to pose the regularization effect; rather, it tries to render the expectation of the squares error of the final identified results minimized. Moreover, in addition to the measurement errors, the model errors in the external load, the damping and even the predefined stiffness of the joint are also considered and how to properly choose the weight matrix in such cases is also addressed.